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NAME OF AUTHOR......Allan R. Buss......

TITLE OF THESIS......Multivariate developmental
psychology: A theoretical
treatise on changes in ability factors.

UNIVERSITY......of Alberta......

DEGREE FOR WHICH THESIS WAS PRESENTED......Ph. D.......

YEAR THIS DEGREE GRANTED......1973......

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THE UNIVERSITY OF ALBERTA

MULTIVARIATE DEVELOPMENTAL PSYCHOLOGY: A THEORETICAL TREATISE ON CHANGES IN ABILITY FACTORS

by

© ALLAN R. BUSS

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF PSYCHOLOGY

EDMONTON, ALBERTA
FALL, 1973

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "Multivariate Developmental Psychology: A Theoretical Treatise on Changes in Ability Factors" submitted by Allan R. Buss in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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Abstract

The broad context in which this dissertation falls is multivariate developmental psychology. Being theoretical and nonempirical in nature, the intent is to explore certain foundational issues such that the area of multivariate developmental psychology may be secured. The central theme involves considering changes in either ability factor loadings, factor scores, and/or interfactor relationships. More specifically, after some general introductory comments in Chapter 1 concerning changes in ability factors, Chapter 2 considers the concepts of quantitative, structural, and quantistructural change as these have been characterized from a multivariate perspective, where the latter is a hybrid of the former two types of change. In Chapter 3 the relationships between learning, transfer, and ability factors are reviewed. After defining transfer and learning within the factor analytic model, a multivariate change model is presented in which changes in ability factor socres (cognitive structure) and factor loadings (task structure) provide the theoretical underpinnings for the process of transfer. In Chapter 4, the relationships between learning, development, and ability factors are considered. This involves integrating Gagné's types of learning with conceptualizations of the development of ability factors. This account provides for a learning stage view of the growth of ability factors. In Chapter 5 three basic ideal factor relation types are outlined, i.e., divergence, convergence, and parallelism. Concepts considered, which provide for an analytic statement as to the possible ontogenetic changes in structure, include cumulation vs. non-cumulation of factors, and higher order

factors. The multivariate ontogenetic change model developed is applied to the classical, differential, and ipsative approaches to developmental change, as well as several developmental concepts. In Chapter 6, the developmental models of Baltes, Cattell, and Schaie are briefly reviewed. Baltes' bifactor model (age and cohort), which involves longitudinal and cross-sectional sequential methods for data gathering, is extended to include a third strategy in the form of the time-lag sequential method. A new possibility for applying the bifactor developmental model to ability factors is considered, i.e., changes in cognitive complexity or the trait pattern (the number of factors and their interrelationships). In Chapter 7, the concepts of inter-individual differences, intra-individual differences, and intra-individual changes are defined in terms of sampling across one of the 3 dimensions of individuals, occasions, and variables respectively. Out of the total of 15 data gathering strategies considered, 11 are all defined in part by the occasion dimension and are therefore capable of dealing with change. In Chapter 8 multivariate approaches to the structuring of behavior are considered with respect to mapping out developmental causal networks. A general recursive-nonrecursive model is presented in which both unidirectional (recursive) and bidirectional (nonrecursive) influence properties are considered. The generality of the model is stressed by noting that the components may assume any of the following: lower order factors, higher order factors, simple variables, and complex criterion variables.

ACKNOWLEDGEMENT

I would like to thank the members of my committee, Drs. J. R. Royce, K. V. Wilson, T. E. Weckowicz, R. S. MacArthur, and P. B. Baltes for their interest and constructive criticisms of this dissertation. Special thanks goes to Dr. Royce and his Center for Advanced Study in Theoretical Psychology. Dr. Royce and his Individuality Project introduced me to an area and an approach to psychology that I did not know existed three years ago. The completion of this project was facilitated largely by his encouragement, stimulation, and freedom granted in pursuing ideas that were personally exciting. Special thanks also goes to Dr. P. B. Baltes, whose work stimulated and shaped much of my thinking. Several other people were instrumental in improving this manuscript. I would like to thank Drs. R. B. Cattell, G. A. Ferguson, J. L. Horn, J. R. Nesselroade, K. F. Riegel, W. W. Rozeboom, and K. W. Schaie for reading some of the chapters. I would also like to acknowledge the partial support of a Canada Council Doctoral Fellowship (W72-4489) and a Canada Council Grant (68-0137) to Dr. J. R. Royce. Finally, I would like to thank Linda Mallett for an excellent typing job.

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CHAPTER 1

INTRODUCTION

Since this manuscript does not fall into the category of a typical dissertation that is carried out within the field of psychology, some general preliminary remarks are in order. The reader will first note that the traditional chapter headings used for organizing an empirical research are absent. Thus there is no general introductory review of the literature that leads up to the formulation of a specific set of empirical hypotheses, followed by sections on method, results, and discussion. Being theoretical and nonempirical in nature, the format adopted here has been to give chapter headings to certain specific issues or aspects involved in changes in abilities from a multivariate developmental perspective. In this way, the pertinent literature for each topic or issue is introduced as required, thereby facilitating the presentation, argument, and understanding of each of the separate chapters. In this sense, each of the chapters forms a semi-autonomous unit, which may stand or fall largely on its own merit.

If the reader comes away with the feeling that each chapter almost reads like a separate paper, this would be essentially a correct conclusion in a certain sense, and is due to the manner in which this dissertation evolved. That is, over the last 12 - 15 months, the writer has been engaged in a self-stimulating process, where after delving into one issue in a relatively comprehensive and analytic manner, related issues and questions became apparent which led to the next chapter, and so on. Thus an initial interest in learning as it relates to ability factors led to a concern with development, which in turn led to several different, but compatible, types of developmental models. While this freedom to follow up interesting leads should also exist in an empirical research, it seems that an exclusive concern with conceptual and theoretical issues provides an added advantage in this regard since one is not restricted by such practical problems as having the available resources to follow one's hunches (e.g. time, financial backing, locating and testing appropriate subjects, etc.).

Having alerted the reader to the distinctiveness of each of the chapters that follows, it should be quickly stated that there truly is a 'thesis' here. The overriding theme or topic is *multivariate developmental psychology*, an area that is beginning to take shape and which perhaps can be considered as being established as an 'area' with Coan's (1966) pioneering chapter. The purpose of the present work is largely to forge ahead in this rapidly expanding field, systematizing and organizing previous work and pointing the way for future developments. Within the broader framework of multivariate developmental psychology, the major focus of attention in the present text is upon *changes* in ability factors. More specifically, *changes in factor scores, factor loadings, and/or interfactor relationships are usually the focus of attention*. It is in this sense that the subject matter dealt with here may be viewed as being rather specific and circumscribed. At the same time, the present treatment may be considered as quite broad in nature because of the several different ways of looking at changes in ability factors that are considered below.

While there has been considerable effort to develop change models within the factor analytic model, it is unfortunate that much of this work has been going on divorced of the more substantive areas in psychology such as (learning and development). Thus, it would seem advantageous to embed the notion of changes in ability factors within the context of those variables that determine change, that is, within a learning and developmental context. This brings us to the important point that the major contribution of the present work is not in the area of developing and extending the more technical aspects of measuring changes in factors, but rather in integrating the concept of changes in ability factors within substantive learning and developmental theory. In other words, the approach taken is a 'model building' one rather than considering the complex problems associated with actually measuring changes in factor scores, factor loadings, and interfactor relationships. In this sense, much of the theoretical work that follows may be considered 'futuristic', since operationalizing many of the models to a completely satisfactory degree depends upon much more sophisticated psychometric and scaling techniques than are presently available. Although these scaling and psychometric problems have been largely bracketed in the main text in order to facilitate the development of the central issues with which the writer is concerned, it would be irresponsible to completely ignore these problems. It is for this reason that a brief summary statement of the problems in detecting 'real' changes in factor scores, factor loadings, and interfactor relationships is presented in Appendix A. Appendix A should probably be read concurrently with Chapter 2, although an understanding of the latter is not dependent upon the former.

Having indicated a major assumption that is unique to the present treatment, a second nonunique assumption should be mentioned that characterizes much multivariate work in general. This is the assumption of linearity, and a brief discussion of this topic may be found in Appendix B. Essentially the idea here is that the factor analytic model is linear and compensatory in nature, unable to detect interactional relationships as well as disjunctive and conjunctive aspects in the data.

In concluding these introductory remarks, it may be helpful to the reader to offer a brief summary statement of the remaining chapters. Chapter 2 sets out the conceptual framework for changes in factor loadings and factor scores. Chapter 3 makes use of the idea of such changes in providing the theoretical underpinnings for the process of transfer. Chapter 4 develops a learning stage theory in the development of ability factors and relates this idea to changes in ability factor loadings and scores. Chapter 5 concentrates on changes in the interfactor relationships within a developmental context. Chapter 6 considers changes in the interfactor relationships within current developmental models that separate age-related and cohort-related effects. Chapter 7 sets out several data gathering strategies for person X variable X occasion data within the familiar concepts of inter-individual differences, intra-individual differences, and intra-individual changes. Finally, Chapter 8 considers recursive and nonrecursive properties of factors and developmental causal networks.

CHAPTER 2

A MULTIVARIATE MODEL OF QUANTITATIVE, STRUCTURAL, AND QUANTISTRUCTURAL ONTOGENETIC CHANGE[1]

The issue of quantitative vs. qualitative (structural) ontogenetic (age-related) change has been a continuing source for discussion in developmental psychology (Flavell and Wohlwill, 1969; Reese and Overton, 1970; Werner, 1957). Recently, a small number of people with a multivariate orientation towards the structuring of age-related behavioral change have focused upon this issue. Thus, Emmerich (1968) has noted the following possibilities as indicating structural change in factors (he uses the term discontinuous): changes in the meaning of a factor as reflected by changes in factor loadings; changes in the number of dimensions required to account for the common variance; changes in the absolute amount of common variance accounted for by a factor; changes in the relative amount of common variance accounted for by a factor; and finally, changes in the intercorrelations among factors. Quantitative change in factors may be identified with what Emmerich calls instability of the rank of individuals on factors, although a prerequisite here would seem to involve no structural change, or what Emmerich calls continuity.

Coan (1966, 1972) has also considered the question of change in factors over the life-span, and outlined several theoretical possibilities for structural change as well as presenting graphs of quantitative change or changes in the level of factors. The former included: factor metamorphosis whereby the factor changes in its basic nature; factor emergence which involves the appearance of a factor due to increasing covariation with age; factor convergence or the coming together of two earlier factors into a single one; factor disintegration which comes about as a result of a decrease in covariance; factor divergence or the splitting of a single factor into two factors; and finally, factor component interchange in which a regrouping of components defining a factor occurs due to changes in covariation.

---

[1] A version of this chapter is to appear in Buss (1973a).

All of the conceptual distinctions that Emmerich and Coan have made with regard to quantitative and structural change in factors must ultimately hinge upon sophisticated technical and psychometric considerations with respect to two concepts. That is to say, it must be possible to determine "real" changes, as opposed to spurious changes, in factor scores and factor loadings across populations (same or different individuals) that are defined by age differences. Bracketing for present purposes the scaling problems associated with determining genuine age-related changes in factor scores and factor loadings (see Appendix A for a brief discussion of these issues), it should be noted that the distinction between multivariate quantitative and structural changes through ontogeny has in fact been recently made within the framework of changes in factor scores and factor loadings (Baltes and Nesselroade, 1970, 1973; Emmerich, 1964, 1966, 1968; Nesselroade, 1970; Nesselroade and Bartsch, 1973). Although the details of these conceptualizations of quantitative and structural change will be examined in greater depth below, it can be mentioned here that the terminology has not always been uniform across investigators. In addition, it will be seen that it is possible to apply the terms quantitative and structural change to a factorially complex or multi-dimensional criterion variable, as opposed to a factor. To compound the complexity of this issue, changes in factor scores and factor loadings have also been used by some of these same people for discriminating between trait and state factors.

The purpose of the present chapter is to offer a conceptual framework that will systematize and extend previous thinking on multivariate ontogenetic quantitative and structural change. It will be seen that different types of change are possible when both the trait-state factor distinction and short versus long term time-spans between the occasions that allow for determining change are co-considered. In addition, the separation of considering changes in factorially complex criterion variables, that span more than one domain (e.g. ability, temperament, motivation), from that of a factor, will be seen to have important implications for generating the total types of possible change through the life-span. Finally, the change concepts developed will be applied to hierarchical factor models.

Changes in Complex Multi-Dimensional Criterion Variables

Before considering specifically changes in complex multi-dimensional criterion variables, it will be advantageous to develop the logic of change within the factor-analytic model. Factor analysis takes as its goal the determination of a relatively small set of factors which permit the generation of a larger set of data. It begins with a set of variables in the form of a symmetric correlation matrix and through a series of mathematical operations (see Harman, 1967; Mulaik, 1972), arrives at a more basic set of latent constructs (factors) revealed by the inherent covariance relationships among the original variables. This procedure permits the specification of an individual's score on any of the variables in terms of the underlying factors. The latter relationship is embodied in what has been called the basic factor equation, or pattern equation, or what Cattell (1957) calls the specification equation. This equation, with the variance associated with uniqueness set aside, may be stated as:

$$z_{xi} = b_{x1}F_{1i} + b_{x2}F_{2i} + \dots + b_{xk}F_{ki} \qquad [1]$$

where z_{xi} is the common part of individual *i*'s standard performance score on variable *x*, the *b*'s are the *k* factor loadings, and the *F*'s are the *k* factor scores. Equation [1] specifies a person's score on a variable as an additive function of his weighted factor scores. The factor scores are unique for individual *i* and remain invariant across any variable considered (no variable subscript), while the factor loadings are unique for variable *x* and remain invariant across any individual considered (no person subscript). The latter statement needs to be qualified by the restriction that it *necessarily* holds only when considering the specification of scores on variables at one point in time, since it is possible that both factor scores and loadings may change over time and thus confound the two invariant relationships just mentioned. The latter possibility is the central issue that is considered in this paper.

A couple of additional comments can be made concerning the relationship expressed in Equation [1]. First, it may be helpful to think of Equation [1] as a special type of linear regression equation where a variable is regressed on factors. Second, the reason for not considering Equation [1] as just another regression equation stems from how the components that are regressed upon, that is, the factors, are viewed. A substantial literature exists on considering factors as valuable theoretical constructs with inferential power, which is the view adopted here (Cattell, 1957, 1966a; Coan, 1964; Guilford, 1967; Henrysson, 1957; Royce, 1963; Rozeboom, 1966; Thurstone, 1947) rather than simply being descriptive components of the immediate data (Burt, 1949; Eysenck & Eysenck, 1969; Thomson, 1956; Vernon, 1950).

Consider now Equation [1] as specifying the performance of a given individual on a given variable at an occasion o_1. It is possible to construct a similar equation for occasion o_2, in which changes on variable x will be a function of either changes in factor scores, factor loadings, or both (Nesselroade, 1970), where it is assumed that the "same" factors are involved (qualifications to be discussed). Consider first the case for changes in factor scores or F's. This situation may be represented by:

$$z^*_{xi} = b_{x1}F^*_{1i} + b_{x2}F^*_{2i} + \dots + b_{xk}F^*_{ki} \qquad [2]$$

where the symbols are the same as in Equation [1] and the asterisk (*) indicates values on occasion o_2 that are different from values on occasion o_1. Equation [1] may now be subtracted from Equation [2] in the following straightforward manner:

$$z^*_{xi} - z_{xi} = b_{x1}(F^*_{1i}-F_{1i}) + b_{x2}(F^*_{2i}-F_{2i}) + \dots + b_{xk}(F^*_{ki}-F_{ki}) \qquad [3]$$

which may be considered as representing *quantitative* change in a multidimensional variable. That is to say, the change in performance in variable x from occasion o_1 to occasion o_2, comes about as a result of increases or decreases on factors whose basic natures are assumed to remain invariant since the factor loadings remain the same. The implication here is that the theoretical underpinnings for quantitative change in a multi-dimensional variable reside in the factor scores (F's), and that for a given individual i, such changes in factor scores will yield quantitative changes across multi-dimensional variables, assuming constant factor loadings for those variables.

Consider now that situation where a change in performance on variable x from occasion o_1 to occasion o_2 is a result of changes in the factor loadings or b's, which may be interpreted as a change in the factor demands of variable x over time. This situation can be represented in a similar manner as was done in Equation [3] for changes in factor scores, that is,

$$z^*_{xi} - z_{xi} = (b^*_{x1} - b_{x1})F_{1i} + (b^*_{x2} - b_{x2})F_{2i} + \dots + (b^*_{xk} - b_{xk})F_{ki} \qquad [4]$$

where the only difference from Equation [3] is that the change in variable x is now due to changes in factor loadings rather than factor scores. Equation [4] represents *structural* change in a multi-dimensional variable, since such change comes about as a result of structural changes in the underlying factor demands that define variable x. This brings us to the conclusion that the theoretical underpinnings for structural change in a multi-dimensional variable reside in the task demands (b's), and as such, will occur across persons for that variable assuming constant F's for other persons. This follows, since it will be recalled that the factor loadings have no person subscript.

The third possibility for change in a multi-dimensional variable involves that situation where both factor scores and factor loadings change across occasions. It is not possible to represent this subtraction of one specification equation from another in terms of the individual subtraction of both component factor scores and factor loadings, since no simple algebraic solution exists when both of these components change. This third possibility may be represented, however, by:

$$z^*_{xi} - z_{xi} = (b^*_{x1}F^*_{1i} + b^*_{x2}F^*_{2i} + \ldots + b^*_{xk}F^*_{ki}) - (b_{x1}F_{1i} + b_{x2}F_{2i} + \ldots + b_{xk}F_{ki}) \quad [5]$$

which simply involves determining each of the separate values on variable *x* before the subtraction operation is carried out. Since this type of change in variable *x* involves both quantitative change (changes in *F*'s) and structural change (changes in *b*'s), it will hereafter be referred to as *quantistructural* change, which will be the only neologism introduced in this chapter.

Several comments are in order on the developments thus far. Mention should be made of the fourth possibility involving no change in factor scores and factor loadings and thus no change in variable *x*. This case will be designated as *stability*. More specifically, by "stability" it is meant that the rank order of individuals' scores across occasions is constant, since it would be possible to have a constant absolute increase in factor scores across individuals, for example, and yet have identical *standardized* factor scores across occasions. If one were to adopt a notion of stability that necessitated constant absolute raw variable scores and factor scores, as well as constant factor loadings, it would be necessary to consider standardizing across occasions, rather than within occasions, the variable scores and factor scores. That is, the combined or pooled scores across occasions would be standardized as a single group. It would also be necessary to consider the factor loadings as *b* coefficients rather than beta weights, since the former are not restandardized within occasions thus avoiding spurious changes. The factor-analytic techniques designed to deal with these problems are reviewed in Appendix A. These four possibilities involving either change or stability for each of factor scores and factor loadings, are represented in Figure 1, where A indicates stability, B - quantitative change, C - structural change, and D - quantistructural change.

Concerning the three types of change outlined above and as represented in Equations [3], [4], and [5], it should be appreciated that logically, any one of them may occur even though there is no change in the manifest score on variable x. This becomes apparent once it is realized that the basic factor equation which specifies performance on variable x is compensatory in nature. That is to say, if there were compensatory (or trading off) changes on whatever component is being considered (factor scores, factor loadings, or both), it is possible that quantitative, structural, or quantistructural change will occur with respect to the source constructs specifying performance on variable x, while the manifest performance score of a particular individual i remains constant (assuming that there are some individuals that do change on some variables, thus altering the factor loadings and/or factor scores). This implies that the zero manifest change on this variable across occasions for a particular individual i is the result of precise compensatory changes, with regard to the original performance score on variable x, in either factor scores, factor loadings, or both. This point leads us to an important conclusion with respect to the types of change on a multi-dimensional variable. Strictly speaking, the distinctions stability and quantitative, structural, and quantistructural

Figure 1

The four change properties of a multidimentional variable.

		F's	
		=	±
b's	=	A STABILITY	B QUANTITATIVE CHANGE
	±	C STRUCTURAL CHANGE	D QUANTISTRUCTURAL CHANGE

change, apply to the latent source determinants of individual differences in variable x rather than the manifest change score of variable x. That is to say, the change properties outlined here refer to changes *in* a multi-dimensional variable (the latent source factors) rather than to changes *on* such variables (the manifest variable score).

The types of change presented above embody previous statements on this issue, with some minor modifications. Thus Nesselroade (1970) identified quantitative change in the same manner as in Equation [3], but considered structural change as reflected by Equation [5] (called here quantistructural) rather than by Equation [4]. Baltes and Nesselroade (1970) have made the same four fold distinctions as in Figure 1 using the terminology stable versus fluctuant factors scores and invariant versus noninvariant factor loadings. They quite correctly argued that quantitative change must have as a prerequisite structural stability, although they do not consider the possibility that structural change should have as a prerequisite quantitative stability. Emmerich (1964, 1966, 1968) has used the terminology stability versus instability for factor scores and continuity versus discontinuity for factor loadings. Also related to the present treatment is Cattell's (1971) trivector structured learning model, where a change in variable x brought about through learning is represented by three vectors that capture changes in the factor loadings, factor scores, and modulating indices respectively. The latter concept refers to differential weights that are assigned to certain concepts (e.g., states, roles, dynamic traits) on the basis of the stimulus situation (see Cattell, 1963, for a discussion of modulation theory).

It will be advantageous to consider the distinctions made above with respect to the types of change possible for the source constructs specifying performance on a variable x, as applying to that situation where variable x is a complex criterion variable that is 'extra' the original factor analysis. Such a move is by no means radical, since there has been considerable work relating source factors in the personality, abilities, and motivational domains, to such performance variables as school achievement (e.g. Cattell and Butcher, 1968) and occupational performance (e.g. Sells, 1966) through the use of multiple correlation and regression techniques. This slight conceptual shift will serve the purpose of clarifying previous treatments of this issue, as well as lay the ground work for subsequent extensions that will be made concerning the change concepts developed thus far. Previous writers (Baltes and Nesselroade, 1970; Emmerich, 1964, 1966, 1968; Nesselroade, 1970) have adopted the framework of considering a multi-dimensional variable that has been included in the original factor analysis. While there can be no criticism of this approach on technical grounds, it seems to the writer that there may be a danger here of overgeneralizing from the correct conclusion that, if there are changes in factor loadings as set out in Equation [4], there will necessarily be structural change in the nature of the factors themselves. While this conclusion must follow for that case where the multi-dimensional variable specified is one of the variables in the original factor analysis and it is assumed that the nature of a factor is reflected by the factor loadings, it by no means necessarily holds for that case where the multi-dimensional variable is 'extra' the original factor analysis.

Concerning that case where the multi-dimensional variable is a complex criterion variable that is 'extra' the original factor analysis, it is conceivable that performance on such a variable $\underline{x}$ on two separate occasions will result in changes in factor loadings while retaining structural invariance with respect to the nature of the factors involved. For example, consider performance in history. Initially, the ability factor memory may be important in producing individual differences and receives a large weight. On a later occasion, one may find that such a factor is of little importance for producing individual differences, but the ability factor deduction may be important. The interpretation here is that initially one needs to memorize 'facts' to do relatively well in history, but at a latter time for the same individual, once a reasonable number of 'facts' have been stored away, relatively high performance now requires the deduction of certain principles, trends, implications, etc. There has been structural change in this example from occasion $\underline{o}_1$ to occasion $\underline{o}_2$ with respect to the degree the source factors are involved in specifying individual differences in performance in history (assuming for the sake of argument no change in the level of factor scores). It by no means follows automatically that the factors themselves as represented by the $\underline{F}$'s have undergone structural change. It is quite possible that the memory and deductive factors are basically the same (structurally invariant) across occasions. The distinctions being hinted at here will become explicit in the following section. Assuming for present purposes then, that the development thus far has been concerned with complex multi-dimensional criterion variables that are 'extra' the original factor analysis, it will now be necessary to consider in greater detail the idea of changes in factors.

Changes in Factors

It is possible to conceptualize performance on a factor in a similar manner as was done in Equation [1] for a multi-dimensional variable. This would be represented by:

$$\hat{F}_{fi} = w_{f1}V_{1i} + w_{f2}V_{2i} + \dots + w_{fj}V_{ji} \qquad [6]$$

where $\hat{F}_{fi}$ is the estimated factor score on factor f for individual i and the w's and V's are the j salient variable loadings and salient variable scores respectively. By salient variables it is meant those variables in the original factor analysis that reflect or specify the intrinsic nature of the factor. Thus in Equation [6], the salient variables are not considered as "determining" individual differences in factor scores, since factors are viewed throughout this paper as the more basic sources of individual differences. What Equation [6] does set out, however, is a definition or the basic nature or meaning of a factor in terms of salient variables. By a similar argument as before, it is possible to specify stability and quantitative, structural, and quantistructural change in terms of change or no change for salient variable scores and loadings. This possibility is set out in Figure 2, where using lower case letters to refer to these distinctions with respect to a factor rather than the previous upper case letters for a multi-dimensional variable, a represents stability, b - quantitative change, c - structural change, and d - quantistructural change. It has already been noted that Coan (1966) has considered quantitative changes in several personality factors over the life-span as well as structural changes in the nature of the factors themselves. Examples of the latter

Figure 2

The four change properties of a factor.

		V's	
		=	±
w's	=	a STABILITY	b QUANTITATIVE CHANGE
	±	c STRUCTURAL CHANGE	d QUANTISTRUCTURAL CHANGE

include the expression of many factors becoming increasingly internalized or more covert with age, as well as structural changes due to the socialization process (e.g. dominance progresses from mere aggressiveness to more socially acceptable assertiveness associated with success).

Reconsider now that situation where equations of the type [1] specify performance on two separate occasions for a complex multi-dimensional criterion variable $\underline{x}$ that is 'extra' the original factor analysis. A little reflection here will make it apparent that, in addition to characterizing the latent source constructs of this equation in terms of the four fold classification of change properties, the factor scores or $\underline{F}$ values in this equation can themselves be similarly characterized as implied by Equation [6] and Figure 2. In other words, in considering the sources for change on a complex multi-dimensional criterion variable (factor scores and factor loadings), one of these two constructs (factor scores) may be further analyzed with respect to change properties. This implies a two-level analysis of change properties for a complex multi-dimensional criterion variable, where the paradigm is: deep change property of a factor → surface change property of a complex multi-dimensional criterion variable → observed change (or no change) on manifest score of complex multi-dimensional criterion variable.

Further comment is necessary regarding the rationale of the two-level analysis of change properties. While it could be argued that any given salient variable could also be considered 'extra' in any particular factor analysis, and therefore lead to an infinite regress, this problem is avoided by the following assumptions and decision criteria. First, it is assumed that the salient variables defining a factor are arrived at via representative sampling of a particular domain (e.g., ability, temperament, motivation). Cattell's (1957) concept of _personality sphere_ is critical here, where one theoretically samples from the sum total of behaviors for each of the ability, temperament, and motivational domains in arriving at a more basic set of source factors. A salient variable, according to this view, would be relatively specific and confined to a single domain. Second, it is important to keep in mind that the variables which are 'extra' the original factor analysis are assumed to be complex criterion performance variables which usually span more than one domain. Given these two assumptions, it would not make conceptual sense to include such cross-domain criterion performance variables in the basic factor-analyses which are focused on identifying the primitive source constructs within each domain. Similarly, to consider a particular highly salient variable as 'extra' the original factor analyses (as defined above), would not be in keeping with the logic developed here, that is, using factor analysis to identify _the_ within domain source factors (in terms of relatively specific salient variables), and then using such factors in multiple regression and correlation procedures for specifying individual differences in more complex criterion performance variables.

Figure 3 sets out the somewhat complex totality of logical combinations of this two-level analysis, where the four change properties characterizing factors interact with (i.e., are coupled with) the four change properties characterizing complex multi-dimensional criterion variables, thus resulting in a $4 \underline{X} 4$ matrix. To simplify matters, it will be advantageous to assume that the change properties characterizing each of the $\underline{F}$'s in a given Equation [1] are identical. This point will

Figure 3

The logical pairings of deep and surface change properties where s indicates stability, i - impossible, and c - change.

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \times \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{array}{c|c|c|c|c|} & A & B & C & D \\ \hline a & s & i & c & i \\ \hline b & c & c & c & c \\ \hline c & c & c & c & c \\ \hline d & c & c & c & c \\ \hline \end{array}$$

receive further consideration in a subsequent section.

Each of the cells of the matrix in Figure 3 are indexed by one of three general properties, where s indicates stability, i - impossible, and c - change. For example, consider that case where the salient variable scores and loadings defining a factor as in Equation [6] are constant across occasions, (which implies constant F's in Equation [1]), as well as constant factor loadings in Equation [1]. The property of this situation ($a \rightarrow A$) may be characterized as stability (s). Two of the cells in Figure 3 are impossible. In considering the second cell of the first row, it would be impossible to have quantitative change for a complex multi-dimensional criterion variable (changes in F's) if the salient variable scores and loadings specifying the factor scores (F's) themselves are constant across occasions. By a similar argument, the fourth cell in the first row would be impossible. One may question the possibility of the second, third, and fourth cell of the first column, where each of these is characterized by having stable factor scores and loadings specifying performance on a complex multi-dimensional criterion variable, and yet have various changes in the salient variable scores and loadings defining the factors. The validity of these three cases follows from the observation that Equation [6], which defines a factor, is also compensatory in nature, and it is logically possible (though perhaps improbable) that changes in either salient variable scores, loadings, or both, will still yield identical manifest factor scores across occasions.

It can be noted in passing at this point that the change properties indicated in Figure 3 would not apply if both the multi-dimensional variable and the factor as set out in Equations [1] and [6] respectively are considered within the same factor analysis. This follows, since the *b*'s in Equation [1] cannot change without the *w*'s in Equation [6] changing, given that the salient variables are a subset of the multi-dimensional variables. In this situation, additional cells in Figure 3 would be impossible, namely, the third cell in row one, the third and fourth cell in row two, and the first and second cell in each of rows three and four. This situation, however, is tangential to the main thrust of this chapter, and will not be further considered.

Table 1 sets out the total number of logical pairings of change properties of factors and complex multi-dimensional criterion variables with their verbal designations, and may serve as a useful reference source during subsequent discussions. Thus it can be seen that only *a*→*A* can be designated by the term "stability", since each of the change property modifiers are "stable". The 13 possible change paradigms are designated "change", which is modified by a deep source change property for factor scores (*a*,*b*,*c*, or *d*) and a more surface change property for the multi-dimensional variable (*A*,*B*,*C*, or *D*. In addition to the 'pure' case of *a*→*A*, there are three other cases of this type, that is, *b*→*B*, *c*→*C*, and *d*→*D*, which would seem to present little difficulty of interpretation. Each of these three 'pure' cases of change involve a deep source change property (quantitative, structural, and quantistructural respectively) that leads to an identical surface change property, such that there is a uniform type of change property that pervades the entire change process.

A selective consideration of some of the more complex change paradigms may serve to highlight some additional issues. The *c*→*B* case, or *structural quantitative* change, implies that surface

Table 1

The Relationship Between Change Properties for a Factor Score and a Multi-Dimensional Variable

Change Properties			
Factor Score	Multi-Dimensional Variable	Change Paradigm	Total Situation
a - stable	A - stable	a→A	stability
b - quantitative	A - stable	b→A	quantitative stable change
c - structural	A - stable	c→A	structural stable change
d - quantistructural	A - stable	d→A	quantistructural stable change
a - stable	B - quantitative	a→B	impossible
b - quantitative	B - quantitative	b→B	quantitative quantitative change
c - structural	B - quantitative	c→B	structural quantitative change
d - quantisturctural	B - quantitative	d→B	quantistructural quantitative change
a - stable	C - structural	a→C	stable structural change
b - quantitative	C - structural	b→C	quantitative structural change
c - structural	C - structural	c→C	structural structural change
d - quantistructural	C - structural	d→C	quantistructural structural change
a - stable	D - quantistructural	a→D	impossible
b - quantitative	D - quantistructural	b→D	quantitative quantistructural change
c - structural	D - quantistructural	c→D	structural quantistructural change
d - quantistructural	D - quantistructural	d→D	quantistructural quantistructural change

quantitative change may be characterized at a deeper level in the change process by structural change. Opposed to this position would be a type of reductionistic stance that would state that, since the deeper sources themselves undergo structural (qualitative) change, this necessitates that what may appear to be quantitative change at the surface level is really structural (qualitative) in nature. Such a position, however, would seem to forfeit the finer discriminations of the total change process that are possible within the present conceptual framework. Also, in considering a similar argument against the b→C case or quantitative structural change, it seems that it would not make conceptual sense to reduce this type of change to simply quantitative change given that structural change occurs at the more surface level. Thus, it would seem advantageous to retain a two-level analysis of the types of change possible in a multi-dimensional variable that is specified by various source factors.

It may be helpful to consider at this point some examples of the change paradigms in Table 1. To simplify matters, consider only one personality factor such as dominance and a complex multi-dimensional criterion variable such as job or school performance. The c→B case or structural quantitative change implies that the dominance factor has undergone structural change (e.g. from overt aggression to socially acceptable assertiveness). This will result in change on the manifest factor score due to a 'redefinition' of the dominance factor. Performance on a somewhat complex criterion variable is now specified by the same amount of dominance (invariant factor loading) although the individual's factor score has changed in level due to a change in the intrinsic components defining this factor. A similar account can be given for the various other change paradigms. In considering ability factors, in Chapter 4 a model will be presented which is thought to provide the theoretical underpinnings for structural change in factors on the basis of different types of learning stages an individual progresses through during ontogeny. In this way, one would have quantitative change in factors within a particular learning stage and structural and/or quantistructural change between learning stages, where the learning stages are identified with Gagné's (1965) seven types of learning (simple S-R to complex problem solving). Change on a complex multi-dimensional criterion variable then, may involve changes in either or both factor scores and factor loadings, where the factors themselves may or may not have undergone quantitative, structural, or quantistructural change with respect to their defining salient variables.

In considering the change properties of both complex multi-dimensional criterion variables and factors, nothing has been said thus far as to the time-span between the occasions used for assessing change. The types of change paradigms possible for short term as opposed to long term time-spans are not identical. In making this further distinction, it will be advantageous to co-consider trait and state factors, since these two types of factors have been cast by some (Baltes & Nesselroade, 1973; Nesselroade & Bartsch, 1973) within the framework being considered.

Traits, States, and Short Versus Long Term Time-Spans

The distinction between trait and state factors has been made by Cattell (1957, 1966b). Traits are considered to be relatively stable dispositional constructs that are trans-situational, that is, they make their appearance in a variety of situations at a fairly stable level. Cattell has considered extensively three different kinds of traits according to modality, that is, ability, temperament, and dynamic traits. This tripartite view of traits has been in the literature for some time, and has been referred to outside the factor framework as cognition, affection and conation respectively, or in more recent times, thinking, emotion, and motivation (Horn, 1966). States may be contrasted with traits in that the former are characterized by being reversible, subject to more rapid changes in level, and modify a different pattern of variables, perhaps even cutting across traits (Cattell, 1966b). State factors typically refer to such things as moods, mental sets, roles, etc. A distinction similar to the trait-state one has also been made by Rozeboom (1965), although his terminology is somewhat different.

Most important in the present context is the distinction between trait and state factors that Baltes and Nesselroade (1973) and Nesselroade and Bartsch (1973) have made within the four fold classification scheme generated from considering change or stability for each of factor scores and factor loadings. Without misrepresenting this view, it will be advantageous to consider it in terms of Equation [6] and Figure 2, and change or stability for salient variable scores and loadings. This is deemed necessary, since in the present scheme, factor scores and loadings were considered for that situation where the multi-dimensional variable was 'extra' the original factor analysis. In terms of Figure 2 then, these investigators consider a trait factor as represented by call a (stable salient variable scores and loadings) and a state factor by cell b (fluctuant salient variable scores and stable salient variable loadings). The rationale for this characterization is that both trait and state factors must have some degree of stability in terms of the salient variables that define their intrinsic nature (the loadings), while a further necessary condition for traits, which have relatively stable levels within persons, must include stable salient variable scores. The latter condition does not hold for states, since they are characterized by fluctuant factor scores within persons and therefore have fluctuant salient variable scores.

The above distinction for trait and state factors is most valuable, but further comment is necessary regarding the generality of this representation. An important point to note here is that this scheme will hold for only relatively *short term* time-spans between occasions, since there is considerable evidence that over relatively long term time-spans, trait factors from the abilities domain, for example, change in level (for reviews see Cattell, 1971, and Horn, 1970), as well as in terms of their basic nature or salient variable loadings (for reviews see Anastasi, 1958, 1970, and Reinert, 1970). The latter list of references refer to that issue of ability factor differentiation over the life-span, where changes in the number of dimensions as well as loading patterns necessitate that the nature of factors undergo structural change. It should be noted in passing that recent discussions of the differentiation hypothesis (e.g. Anastasi, 1958; Reinert, 1970) have pointed out the serious methodological problems involved in detecting genuine age-related differentiation of ability factors. Although there is as yet no empirical evidence for changes in the nature of state factors over the life-span, such a possibility seems reasonable to entertain.

In considering the total change situation for a complex multi-dimensional criterion variable within the context of relatively short term time-spans, it becomes apparent that only a limited number of change paradigms from Table 1 are permissible to the extent one is considering trait and/or state factors as these have been characterized by Baltes and Nesselroade (1973) and Nesselroade and Bartsch (1973). It should be appreciated that this question may be answered *a priori* or on logical rather than on empirical grounds given the definitions of trait and state factors. Consider first then, that situation where a complex multi-dimensional criterion variable is specified by factors that are all traits. Since traits are characterized by cell *a* across occasions, only four change paradigms may be considered, namely, *a*→*A*, *a*→*B*, *a*→*C*, and *a*→*D*. Since the second and fourth change paradigms are impossible, and the first is actually stability, this leaves only the third change paradigm that may occur over relatively short term time-spans between occasions when all the factors specifying a complex multi-dimensional criterion variable are traits. If all the factors are states rather than traits, this implies that one is restricted to the following subset of change paradigms: *b*→A, *b*→B, *b*→C, and *b*→D. To the extent that both trait and state factors are involved in specifying performance on a complex multi-dimensional criterion variable, one will have a totality of five change paradigms available plus the possibility of stability. In considering that situation where both trait and state factors co-exist in a given specification equation, it becomes apparent that this will preclude the possibility of a single change paradigm applying uniformly. That is to say, while the surface change property will apply to the entire specification equation (*A*, *B*, *C*, or *D*), change properties of deeper sources for change (a, b, c, or d) will apply individually to each factor in Equation [1]. This is not to say, however, that there would be no situations where a single change paradigm would characterize the entire two-level change process, and such possible cases may include that situation where ability factors specify performance on a complex multi-dimensional criterion variable and are all in the same learning stage (Buss, 1973b). In this situation, it is possible that all the factors undergo quantitative change if occasion o_1 and occasion o_2 are within the same learning stage, or structural or quantistructural change if the occasions span more than one learning stage. Regardless of whether a single or several change paradigms apply to a particular specification equation over relatively short term time-spans between occasions, the trait-state distinction implies that the deeper source change properties in such situations must be either stability (a) or quantitative change (b).

In considering relatively long term time-spans between occasions, since the possibility has already been noted that both traits and states may change in their basic nature as well as in their level over the life-span, one may add to the previous a and b cases for trait and state factors respectively, cases c and d. Since both the c and d cases apply to both trait and state factors when considered over relatively long term time-spans between occasions, it is not necessary to discriminate between these two types of factors for the purposes of outlining the possible types of change in this situation.

Although any of the previous change paradigms applicable to relatively short term time-spans between occasions would also be possible for relatively long term time-spans, in the latter situation one would expect a preponderance of c and d cases for deeper source change properties. (Incidentally, this question would be answered on empirical rather than on logical grounds.) That is to say, in considering life-span changes in performance on complex multi-dimensional criterion variables, the factors will themselves undergo structural (c) as well as quantistructural (d) change. In any case, the entire armament of possible change paradigms in Table 1 is available in considering relatively long term time-spans between occasions. This model may be made more general by considering m occasions (thus yielding m-1 two-level change paradigms), where there may be a relatively short or long interoccasion interlude for specifying relatively short versus relatively long term time-spans respectively.

Hierarchical Factor Models and Change

A second situation in which it may be profitable to apply a two-level analysis of change involves a consideration of hierarchical factor models, where one maps out a hierarchical structure of a given domain by factoring the primary or first-order factors to yield more general second-order factors. Higher-order factor analysis has been typically carried out for traits within the ability, temperament, and motivation domain. Within the Cattellian framework, higher-order factors are viewed as more general sources producing individual differences in the primary traits, or, as Royce (1963) notes, are embedded deeper within a nomological net and thus are potentially broad explanatory theoretical constructs. This view may be characterized by considering the factor f in Equation [6] as being defined by higher-order factor loadings and factor scores rather than by the salient variable loadings and scores. That is, within a hierarchical factor model, one may view the higher-order factors as sources producing individual differences in the primary factors, which in turn are sources producing individual differences in the variables.

In the present context, the model expressed in Equation [1] may be viewed as either specifying a complex multi-dimensional variable that is 'extra' the original factor analysis, or a more narrow multi-dimensional variable that is part of the original factor analysis. By a similar argument as before, a two-level analysis of change is possible, where the more surface change property is modified by a more deeper change property. Table 1 may again be viewed in terms of setting out the change properties of a factor score and a multi-dimensional variable, where individual differences in the former are now specified in terms of higher-order factors, and the latter may or may not be 'extra' the original factor analysis. The general case would involve $\underline{n}$-order factors (and thus $\underline{n}$ change properties characterizing a given two-occasion change), and $\underline{m}$ occasions (and thus $\underline{m}$-1 sequential sets of $\underline{n}$ change properties).

Having set out the general framework for viewing changes in factor scores and factor loadings over the life-span, the next two chapters will focus upon the topic of learning as it relates to such changes. In Chapter 3, both changes in factor scores and factor loadings in ability factors are considered within the context of learning, and are seen to provide the theoretical underpinnings for the process of transfer. Complementing the somewhat formal model that is developed there, Chapter 4 considers substantive learning theory, or more specifically, basic learning principles as they relate to changes in factor scores and factor loadings.

CHAPTER 3

LEARNING, TRANSFER, AND CHANGES IN ABILITY FACTORS: A MULTIVARIATE MODEL[2]

That there is a need for an adequate theoretical integration of the topics of human abilities and learning has been noted by several prominent leaders in the field. Jensen (1967, p. 131) has stated that "one of the major tasks of differential and experimental psychology is the theoretical integration of individual differences in learning and the structure of mental abilities as represented by tests like the Primary Mental Abilities." Eysenck (1967, p. 87) makes a similar plea when he advocates "that we should take seriously the theory relating the concept of 'intelligence' to learning efficiency and speed ... it seems reasonable to expect that such investigations are more likely to help in the elucidation of the nature of intellectual functioning than is the continued construction of IQ tests of a kind that has not materially altered in fifty years." In the same year that these two statements appeared, Guilford (1967) came out with his important book, The Nature of Human Intelligence, which attempted to put the study of human abilities within the mainstream of general experimental and theoretical psychology.

In considering the relationship of human abilities and learning, there are three distinct aspects upon which one may focus. First, the role of already acquired abilities in the learning of a specific task may be examined. This problem has been extensively studied by Fleishman (1954, 1967) and Fleishman and Hempel (1955) who have carried out certain aspects of Ferguson's (1954, 1956) theorizing. This line of research involves the mapping out of task demands in terms of the sequential changing role of various ability factors. The assumption made here is that ability factors are relatively stable organismic variables that are called into play in varying amounts during the acquisition of a specific skill. Throughout the training process, the organismic variable that is assumed to change is the specific skill rather than or including the ability factor scores. Corballis (1965) has questioned this assumption and argues for an alternate interpretation of Fleishman's results, namely, that the factor loadings remain constant throughout the task and the factor scores are changing. This interpretation would be congenial with Rozeboom's (1971) recent change model. However, the possibility remains that both the factor loadings and factor scores are changing, a possibility which is examined below. It should be noted that Fleishman's paradigm has recently been applied to concept learning (Dunham, Guilford, and Hoepfer, 1966) and verbal tasks (Fredricksen, 1969). The latter study makes use of the notion of cognitive strategies as accounting for transfer effects, since the implementation of a given cognitive strategy across several tasks facilitates performance on later occurring tasks which makes use of this same cognitive strategy.

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[2] A version of this chapter appeared in Buss (1973b).

A recent conceptual linking of abilities and learning has been completed by Merrifield (1966). What Merrifield did was to list Gagné's (1965) eight types of learning (from simple S-R to the more complex problem solving) and 'plugged in' some of Guilford's (1967) 120 structure of intellect factors where they would be expected to play some role. This synthesis of human abilities and learning is a crude first approximation that is of the same formal type as Fleishman's experimental efforts, namely, the role of already acquired abilities in a learning situation.

A second consideration of the relationship of human abilities and learning revolves around the question of the role of learning in the acquisition of ability factors. Ferguson's (1954, 1956) transfer model was one of the earliest formulations of this problem and receives further examination below. Briefly, it was his contention that an ability factor reaches its asymptote and achieves stability through overlearning. The asymptote for a particular individual at a particular point in time is set by heredity and maturation. It is the learning of specific acquisitions which transfer and thus provide for a relatively broad ability factor. Intellectual development viewed as cumulative learning has received attention from Gagné (1965, 1968a, 1968b), Hunt (1961), and Stinchcombe (1969). Gagné's formulation is potentially the most analytic in that he provides for different types of learning (simple to complex) as well as putting to work such notions as transfer learning, hierarchies, and "complex and interacting structures of learned capabilities [Gagné, 1968a, p. 190]." Valuable gains may be in store if one were to rework Gagné's model in terms of ability factors.

A formulation similar to Gagné's, yet going beyond it in terms of relating learning to the growth of an ability factor, has been advanced by Whiteman (1964). In this paper, ability factors, Harlow's learning sets, and Piaget's operations are systematically compared and similarities noted in terms of their intersituational consistency and hierarchical organization. Implications for enrichment (learning) procedures are discussed in terms of this tripartite view of intelligence.

In a similar vein, Goulet (1970) has attempted to integrate various developmental models with respect to the training of transfer. Most relevant here is that after noting both specific (associative) and nonspecific (nonassociative) sources of transfer, the Piagetian training research is reviewed and interpreted in terms of Harlow's learning sets. Since a learning set is considered as a nonspecific source of transfer, Piagetian operations are considered to be broad, trainable, intellectual tools. The trainability of Piagetian operations is generally accepted, although the degree to which nonspecific transfer occurs has not yet been settled (Brainerd and Allen, 1971; Butcher, 1968; Hunt, 1969). To the extent that Whiteman (1964) is correct in stressing the conceptual similarities of a factor, Piagetian operation, and learning set, then the possibility of training ability factors is a reasonable hypothesis to entertain. Indeed, direct evidence of the trainability of ability factors is beginning to emerge. Guilford (1967) has reviewed quite extensively the positive findings regarding the training of divergent factors. Also supporting the trainability of ability factors within the Guilford model is a study by Jacobs and Vandeventer (1972). In a review of this area, Ferguson (1965) cites the work of one of his students (Sullivan, 1964) which supports the trainability or transfer interpretation of abilities. All this evidence would seem to cast doubt on Fleishman's assumption that ability factor scores remain invariant during practise on a task which involves these same factors. To the extent that Fleishman has typically employed young adults in his research, where ability factors are at or near their life-span asymptote, he has some defence of his position, although a model which allows for the possibility of both changes in factor scores and factor loadings would more adequately resolve this issue.

The third consideration of the relationship between human abilities and learning concerns the effects of learning on the organization of ability factors (inter-factor relationships). This important problem will only be briefly mentioned for the sake of completeness since it is not of direct concern to this chapter. Ferguson (1954, 1956) has interpreted the positive manifold of ability factors as resulting from the transfer process. The early differentiation hypothesis (Burt, 1919, 1954; Garrett, 1938, 1946) of the ontogenetic organization of factors was interpreted in terms of maturation. Recently, cultural-learning considerations regarding the ontogenetic organization of ability factors has received attention (Anastasi, 1970; Cattell, 1971; Horn, 1968; Vernon, 1969). The general issue of changes in the organization of ability factors over the life-span is considered in detail in Chapter 5.

The present chapter is concerned with the development of a change model for ability factors that accommodates both the role of ability factors in learning a specific task and the role of learning in the acquisition of a factor. After discussing in general terms the relationship of learning, transfer, change, and ability factors, as construed within the factor analytic model, a change model for learning affecting ability factor scores is presented. This change model will provide for a re-examination of Ferguson's transfer model and will lead to a more complete theoretical statement as to the nature of transfer. Since the learning of a specific task is defined below in terms of relevant ability factors, and changes in the factor scores of these ability factors brought about through learning the task are permitted, it can be appreciated that it is the first two types of relationships between learning and ability factors as outlined above that are of present concern.

Factors, Transfer, and Learning

It is possible to conceptualize within a factor analytic framework both the process of transfer and changes in ability factors brought about through learning a specific task. Consider Equation [1] as specifying a learning task x performance score rather than a test variable score. If one identifies performance on task x as occurring on occasion o_1, and there is a learning situation where practice on a different task y occurs on occasion o_2, it is possible to specify the performance on task y on occasion o_2, by a similar equation, namely:

$$z^*_{yi} = b^*_{y\alpha}F^*_{\alpha i} + b^*_{y\beta}F^*_{\beta i} + \ldots b^*_{yk}F^*_{ki} \qquad [6]$$

where z^*_{yi} refers to the standardized performance score of individual i on task y after practice on both task x and task y. One can then define transfer as the difference in performance on task y as a consequence of practice on task x as opposed to no practice on task x. The asterisk (*) in [6] indicates values that would normally be different from [1]. Besides the obvious differences in task performance score, there are two possible types of change in structure: (*a*) change in *cognitive structure* (the factor scores); and (*b*) change in *task structure* (the factor loadings). Related to this view is Cattell's (1971) attempt to formalize changes in factor scores, factor loadings, and modulated motivational traits in a *structured learning model*. These three types of change are represented by his *three vector change model*, in which a learning change from occasion $\underline{o}_1$ to occasion $\underline{o}_2$ is translated into three vectors respectively, capturing the component changes which define that which occurs on left side of the specification equation.

The amount of transfer which takes place from task $\underline{x}$ to task $\underline{y}$ is considered to be intimately related to the above types of change in structure. If practice on task $\underline{x}$ results in changes for an individual on certain ability factor scores which are involved in task $\underline{y}$, then performance on task $\underline{y}$ will be in part a consequence of such changes (transfer). The more similar task $\underline{x}$ is to task $\underline{y}$, the greater the transfer effects. If task $\underline{x}$ is identical to task $\underline{y}$ in terms of task structure, then transfer in this case reduces to learning one task.

In considering changes in ability factor scores (cognitive structure) that are brought about through learning a specific task, it is necessary to define learning within the framework of ability factors as opposed to behavioral responses. Accordingly, learning will be defined in terms of *relatively permanent changes in ability factor scores* (Royce, 1973a). There are several points to be made concerning this view. First, there is no intention to restrict all of learning to changes in ability factors instead of behavior. Thus the learning of much incidental material (e.g. paired associate learning) will not be interpretable within the present context. This brings us to the important point that in considering the relationship of learning and ability factors, the present focus is specifically upon ability learning (or the learning of abilities) rather than defining *any* instance of learning as changes in ability factor scores. A second point to note is that learning is defined here in terms of relatively permanent changes in ability factor scores or *traits* rather than *states*. Since state factors are subject to greater fluctuation than trait factors, the former may be considered as intimately associated with performance rather than learning. Thus state factor scores such as moods will affect performance, but if learning occurs, this will be reflected in terms of changes in trait factors.

A third point to note here concerns the ontological status of the concept "factor". As mentioned in the previous chapter, the position taken here is that ability factors are useful theoretical constructs rather than simply being a useful method for data reduction, or principles of classification or description. Justification for the former view has been made on the basis of rotating factors according to some criteria (e.g. simple structure) such that they may be meaningfully interpreted, as well as the possibility of establishing some degree of factor invariance across either persons, variables, time, or some combination. Henrysson (1957), as well as Rozeboom (1966), have identified factors extracted from common variance as hypothetical constructs with inferential power, while principle component factors derived from the total variance are thought to represent intervening variables. Both hypothetical constructs and intervening variables have been discussed by MacCorquodale and Meehl (1948) and Rozeboom (1956). The former are characterized by having "surplus meaning" and some generalizability, while the latter are simply a short hand description of the immediate data. It would strengthen Henrysson's argument for factors as hypothetical constructs by adding that in the common factor case, there should also be representative sampling of the variables in the particular domain of inquiry, a point made by Cattell (1957) in that one should ideally sample from all of the variables which represent a particular aspect of the "personality sphere".

The more conservative British position in regard to the status of factors has been characterized by exalting on the dangers of "reifying" factors. As Coan (1964) has noted in a comprehensive treatment of this issue, those that consider factors as theoretical constructs (e.g. traits, attributes, hypothetical constructs, dispositional constructs) never meant by this that they were "real" in the sense that they were "things" that could be located in space and had an existence apart from the behavior in which they were expressed (reification). They were considered "real" in the sense that they are psychologically real, i.e., abstractions from behavior. This brings us to the third point concerning learning and changes in ability factors. The learning concept "habit" may be considered as a hypothetical construct and as dispositional in nature. Inferences concerning such theoretical constructs in terms of learning (e.g. strengthening) are of course made on the basis of a data base (behavior). Similarly, given the view that an ability factor is a hypothetical construct (Henrysson, 1957) and dispositional in nature, it seems quite legitimate to consider that learning may affect such a construct as inferred from changes in behavior (performance on ability tests), where the changes are a result of experience or practice. The fourth point to be made here concerns a somewhat finer conceptual matter. Rozeboom (1965) has correctly noted that learning involves a change in relatively permanent dispositional or trans-situational variables[3] rather than behavior per se, since a little reflection will reveal that behavior is usually never relatively permanent and varies from moment to moment as a function of conditions in and out of the organism. This observation leads to considering relatively permanent properties of an organism brought about by changes in ability factors (as a consequence of experience or practice), as that which is meant by learning.[4]

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[3] Rozeboom uses the terminology state vs. process variables. A state variable is relatively permanent as compared to the short-term process variable.

Finally, it should be noted that in adopting the view of changes in ability factors as that which is meant by learning, it becomes necessary to reconsider how an ability factor may be viewed as being strengthened as this concept is applied to say habits. When a habit such as bar pressing becomes strengthened, this means that under appropriate stimulus conditions bar pressing behavior increases in frequency, has a high probability of occurrence, has a low latency, is resistant to extinction, etc. Whatever measure is used for inferring the strength of the habit bar pressing, one thing remains constant, i.e., the behavioral response which provides for inferring the underlying habit. When one considers the effects of learning on abilities, it is clear that changes in the scores of the underlying constructs provide for different forms of behavior; e.g., increases in the ability numerical reasoning do not merely strengthen early behavior, but allow for higher level cognitive tools to be brought to bear on a problem. It is thus possible to conceptualize an ability factor as being strengthened in two distinct ways. First, where there is no change occurring in the level of an ability factor, the underlying construct may be strengthened according to such criteria as response frequency, latency, probability, and resistance to extinction. In this case, the behavioral response under consideration remains the same and is analogous to strengthening a habit. The second manner in which an ability may be considered to be strengthened is when the potential level of the ability is yet to be realized and learning mediates changes in the factor score. What is being strengthened in this case is the underlying potential level of the ability. Different forms of behavior that can be evaluated along the dimension high-low or good-bad performance (which is not possible, say, in the personality domain) allow for inferring changes in the underlying ability. When these changes are in the direction of closing the gap between present performance level and potential performance level, it is appropriate to speak of strengthening the potential performance level as opposed to strengthening the present performance level. When learning mediates changes in abilities that result in a lower level of performance, this may be viewed as a weakening of both potential performance level and the previous present performance level, but of strengthening the now present (lower) performance level.

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[4] Throughout this discussion, I am dealing with learning effecting the growth of an ability factor. Recognition is hereby given to the limits which an ability factor can reach as determined by individual differences in genetic endowment. However, the latter important determining influence still leaves open the nature of the learning process whereby that inherent limit is reached.

Learning and Changes in Cognitive Structure

Consider now a situation where there is a change in ability factor scores from occasion o_1 to occasion o_2 as a result of practice on task x, i.e., quantitative change as represented by Equation [3] in Chapter 2. In matrix notation, Equation [3] may be expressed as follows:

$$Z^* - Z = A(F^*-F) \qquad [7]$$

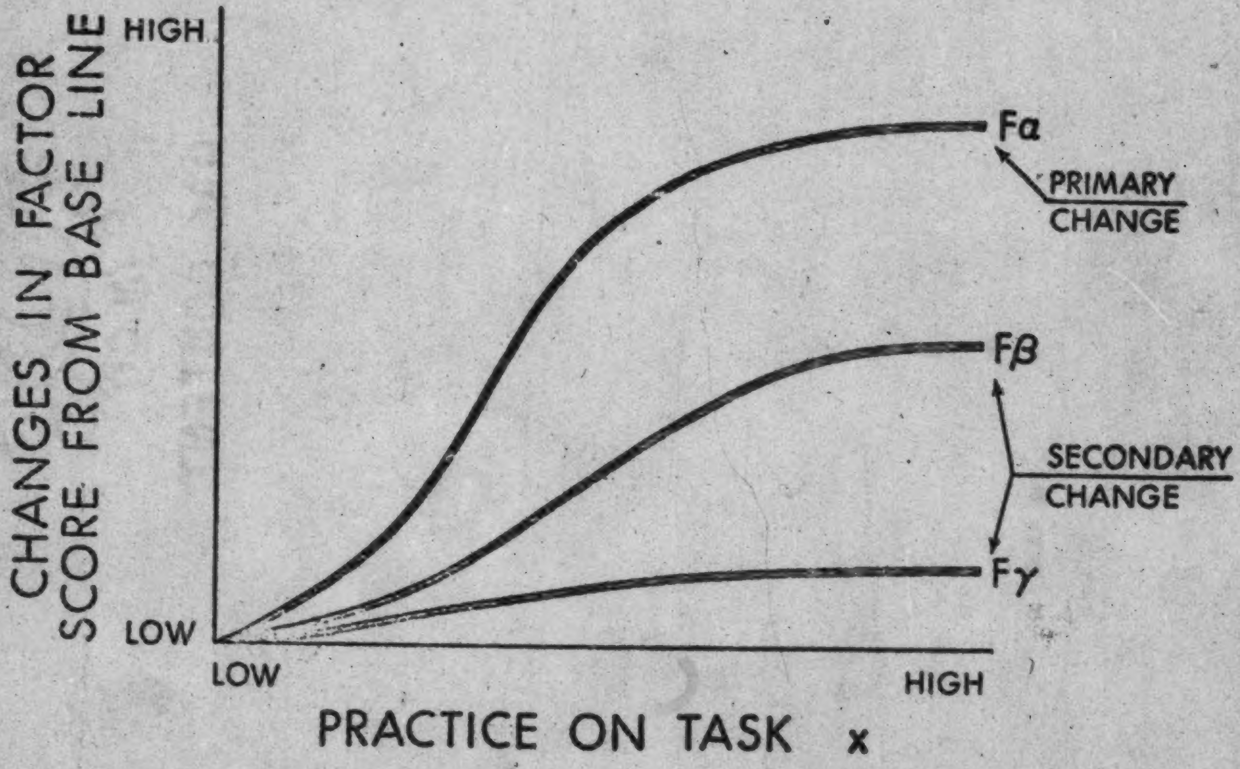
where Z is the n variable by N person score matrix, A the n variable by k factor factor pattern matrix, F the k factor by N person factor score matrix, and the asterisk indicates a matrix at a latter occasion. Let us now define a k factor by N person T matrix as the matrix of total change scores in factors, i.e.

$$T = F^* - F \qquad [8]$$

Consider now the possibility of decomposing each total change score from matrix T into two distinct components. It will be advantageous to consider first the simple case where only ability factor α is operative in practice on a task x and a universe of discourse consisting of factors α,β, and γ. The two distinct types of change in ability factors are primary change (Δ_p) on the operative factor α and secondary change (Δ_s) on the nonoperative factors β and γ. The latter type of change may be thought of as changes in nonoperative factors that are brought about by generalization effects of primary change in the operative factor. Such generalization effects may be specified by a multiplicative function of the psychological structural similarity coefficient (p) of the operative and nonoperative factors, and the primary change of the operative factor ($p\Delta_p$). Figure 4 illustrates these relationships where the nonoperative factors β and γ have different p coefficients with the operative factor α. For example, performance on task x may involve only the ability factor spatial relations (factor α) and not reasoning (factor β) or perceptual speed (factor y). If after practice on task x an increase in the spatial relations factor occurs, one may expect that there will

Figure 4

Primary and secondary changes in factor scores in the simple case where factor α alone is operative in practice on task x and the psychological structural similarity coefficients between the factors α, β, and γ are such that $p_{\alpha\beta}$ is high and $p_{\alpha\gamma}$ is low.



be generalization effects such that there will be minor changes in the nonoperative reasoning and perceptual speed factors. The latter secondary changes will be a function of the primary change on spatial relations as well as the psychological similarity of reasoning and perceptual speed to spatial relations.

In considering the more general case for k factors, where some may be operative and some nonoperative, the total change (Δ_t) for any factor will be equal to the primary plus the various secondary change components i.e.

$$\Delta_t = \Delta_p + \Delta_s \qquad [9]$$

where for nonoperative factors the primary change component (Δ_p) would drop out. The primary change for any factor may be actually considered as a special case of secondary change, i.e., the p coefficient for that factor with itself (which equals unity) matched multiplicatively with the primary change for that factor. This formal property allows one to express [9] much more cogently in matrix notation i.e.

$$T = PC \qquad [10]$$

where T is the matrix of total change scores for k factors and N individuals, P is the matrix of psychological structural similarity coefficients (weights) for k factors and is symmetric with unities in the diagonal, and C is the matrix of primary change scores for k factors and N individuals. What [10] sets out is the basis for a general multivariate change model that provides for teasing out the primary and/or secondary changes on ability factors in a learning situation. As Figure 4 reveals, changes in factor scores increase as a function of practice[5] until an asymptote is reached. Change functions for each ability factor could theoretically be established for various tasks.

Further comment on the psychological structural similarity coefficients from the P matrix is in order. The p coefficients are viewed as invariant parameters not subject to changes brought about by changes in score distributions. This is one feature which distinguishes the p coefficient from the correlation coefficient. A correlation between two traits can be brought about by sources other than the psychological structural similarity aspects (Anastasi, 1970; Thompson, 1957, 1966; Tryon, 1935) although Tryon (1935) considered the similarity of psychological components source as the most potent in producing correlations between variables. To the extent which this is the case one may use correlations as given in an $R_{\underline{pq}}$ matrix of interfactor correlations as crude approximations of the p coefficients if an additional precautionary measure is taken, namely, obtaining estimates of correlations as relatively invariant population parameters by the use of averaging techniques (Guilford, 1965). This procedure will allow for estimating the primary change scores by:

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[5] Throughout the development of this paper, practice is construed as occurring across occasions, i.e., $\underline{o}_1 + \underline{o}_2 + \ldots + \underline{o}_m$. Trials $\underline{t}_1$-$\underline{t}_n$ are collinear with occasions $\underline{o}_1$-$\underline{o}_m$. Although the word 'trial' usually connotes rather short intervals between occasions (i.e., a typical experimental laboratory task), this terminology is intended to be much broader in conception. In other words, since no restriction is placed on the time interval between trials or occasions, the model being developed here is applicable and interpretable from the standpoint of experiences through the life-span (i.e. learning and development). A similar argument can be made for the length of a trial, i.e. ranging from a few seconds or minutes in the laboratory to a fairly long complex life experience.

$$C = P^{-1}T \qquad [11]$$

which follows from [10]. In this way one may generate both primary and secondary changes since, besides the estimates of the p coefficients, one would also have the total change scores.

The three types of change encountered above must be conceptually distinguished since they are not all of the same logical type. Total change is that change which is manifested empirically. It is at the terminal end of the change process. Primary and secondary change lie casually deeper in the change process and must be inferred from considerations other than simply measuring terminal change, since the latter is identified with total change. Although primary and secondary change are conceptually distinct, as indicated above, they may be regarded as being of the same formal type (secondary change), since primary change shares the important property of secondary change of having a psychological structural similarity coefficient, which is defined in this case for a given factor f with respect to itself.

In concluding this section, it can be noted that the change model for ability factors that has been developed here makes no commitment to a particular learning theory. Such substantive considerations are developed in the next chapter.

Transfer and Changes in Cognitive and Task Structure

This section will examine the process of transfer from task x to task y by making use of the preceding model. Transfer occurs when practice on task x has an effect on the performance of task y. One of the earliest attempts to provide a model which ties human ability factors to the process of transfer was that of Ferguson (1954, 1956). The position taken here is that the basis for the process of transfer from task $\underline{x}$ to task $\underline{y}$ resides in part in changes in ability factor scores as a result of practice on task $\underline{x}$. In considering Ferguson's transfer model, it will be seen that it needs to be extended somewhat in order that one may become more analytic as to how transfer occurs as a result of changes in ability factor scores. The model states that performance on task $\underline{y}$ is some unspecified function of performance on another task $\underline{x}$ and the amount of practice on the two tasks $(\underline{t_x}, \underline{t_y})$, i.e. $\underline{y} = \phi(\underline{x}, \underline{t_x} \underline{t_y})$. In the case where practice is defined in terms of performance, then the transfer model simplifies to $\underline{y} = \phi(\underline{x})$, i.e. performance on task $\underline{y}$ is some unspecified function of performance on task $\underline{x}$. An ability is conceived here as evolving out of differential transfer situations and reaches a crude stability of invariance through overlearning. Thus individual differences in abilities reflect a crude limit of performance reached by overlearning. The positive correlations between human abilities are also explained by alluding to the process of transfer. Transfer is thus construed as providing the basis for the establishment of a general factor, since reciprocal influences among primary factors allow for higher order factors to evolve from lower order structures. However, as Whiteman (1964) has noted, this process is complicated since generalization (higher order factors) may also occur because of lack of cognitive differentiation, which is the major tenet of the differentiation hypothesis (Burt, 1919, 1954; Garrett, 1938, 1946).

It may be appreciated by now that Ferguson had a multi-view of the process of transfer, in which the various meanings of this process were not adequately delineated or contrasted. Besides developing the notion that transfer occurs from task x to task y (the one and only view adopted in this chapter), Ferguson stated that transfer provides for the growth of ability factors, is the basis for positive correlations between abilities, and occurs from abilities to learning tasks. In considering the growth (changes) of ability factors, Ferguson does not specify how transfer yields an ability factor. An ability factor is said to emerge out of differential transfer situations. The exact nature of how this factor emerges is left unspecified. Transfer occurs from task to task. How exactly this relates to an ability factor is not indicated. In order to clarify and meet the objections being made above, it is necessary to conceptualize the growth of an ability factor as a result of practice on a task which leads to changes in the underlying factor. Practice on different tasks results in varying changes on different factors. The implication of this view is that transfer from task x to task y is mediated by the changes in factor scores that are brought about by practice on task x. The paradigm is as follows: practice on task x → change in factor scores → transfer → practice on task y.

It is now possible to obtain a clearer idea as to how transfer from task x to task y is effected by changes in the ability factor scores by applying the multivariate change model as expressed in [10]. Consider a situation where changes in ability factors are obtained at the end of performance on task x, and practice, and thus transfer to task y, is to take place. It then becomes apparent that the three types of changes in ability factors as a result of practice on task x form the basis for three corresponding types of transfer. Thus total change provides the basis for total transfer, primary change is the basis for primary transfer, and secondary change is the basis for secondary transfer.

It is important to bear in mind the scope as well as the restrictions of the transfer model that has been presented. In terms of scope, the above model provides for a general multivariate approach in mapping out changes in factor scores as a result of learning (practice on task *x*). The theoretical basis for the process of transfer from task *x* to task *y* has been established, namely, the separate factor score changes that provide for both primary and secondary transfer. This formulation allowed for a general model to get at the basis for total transfer after practice on task *x*. The development thus far has not commented on exactly how much transfer will occur from task *x* to task *y*. That is to say, the model above represents the *basis* for the maximum *possible* transfer effects as a result of practice on task *x*. How much transfer will occur from task *x* to task *y* depends upon: (*a*) the changes in factor scores as a result of practice on task *x*, and (*b*) the nature of task *y*, i.e. the factor structure of task *y* in terms of the factor pattern coefficients (task demands). The model developed thus far has been concerned with the first aspect of this question of possible transfer effects. The second aspect in considering the possible transfer effects, will be a function of the similarity of the factor structure of task *x* and task *y*.

It is now desirable to derive a coefficient or index of the similarity of the factor structure for task *x* and task *y* based upon the factor pattern coefficients in order to determine the *actual* amount of transfer that will take place. Given the factor models specifying performance on task *x* and task *y* (equations [1] and [6]), a coefficient of the similarity of task structures can be obtained using Cattell's (1957) *pattern similarity coefficient* (r_p). As noted by Bolz (1972), the r_p is vastly superior to the correlation coefficient as well as 27 additional similarity or distance measures that have been proposed if it is important to simultaneously consider the elevation, scatter, and shape of component measures. In evaluating the similarity of two task structures, it would be important to simultaneously consider the elevation, scatter, and shape. Of special significance of r_p is that it has a value between ±1 and Horn (1961) has worked out the distribution allowing for evaluating its degree of significance.

The introduction of the r_p coefficient has been preliminary to the aim of linking up the maximum possible transfer as determined by the two separate types of change in structure (cognitive and task) in going from practice on task x to practice on task y. It will be remembered that the maximum transfer possible from task x to task y was considered a function of the total change in ability factor scores (primary plus secondary change) as a result of practice on task x, and the extent to which task x and task y had similar task factor structures (b loadings). This idea may be represented by

$$\text{transfer } x \to y = \phi r_p, T \qquad [12]$$

that is, transfer from task x to task y is some function ϕ of the similarity of the task structures (r_p) as well as the total changes in ability factors resulting from practice on task x. In passing it can be noted that it may be desirable to differentially weight the factor scores in T, since those factors that are important for task y (i.e. high factor loadings) should be given greater weight than those not important (i.e. low factor loadings).

Equation [12] represents the effective transfer performance score from task x to task y for N individuals. The derivation of this model has allowed for an understanding of the complex nature of the transfer process to task y as a function of practice on task x. It may prove enlightening to offer a very simple explanation of the effective transfer performance score for transfer from task x to task y for individual i. It is simply the difference in performance on task y for individual i under two conditions: practice on task x and no practice on task x.

Discussion

The transfer model under consideration receives support from empirical findings that were reported from Heinonen (1962). He investigated the question of whether practice on a marker task for a certain factor would have transfer effects to performance on marker tasks of two other factors. He hypothesized that transfer effects would be an inverse function of the angular separations in common factor space of the marker task vector on which training was received and the marker task vectors on which transfer was measured. In other words, since the correlation of two task vectors is inversely proportional to their angular separation, Heinonen was predicting that the greater the correlation of task x and task y, where task x is the practice task and task y is the task on which transfer occurs, the greater the transfer effects. The hypothesis was confirmed since the increase in mean performance for a task varied systematically with the angular separation of that task from the practice task. Although this investigation does not directly test the notion of primary and secondary transfer effects having their basis in primary and secondary changes in factor scores, these results are not inconsistent with the complete transfer model developed in this paper. This investigation also provides evidence for the plausibility of using correlation coefficients as crude approximations of the psychological structural similarity coefficients.

In considering the kind of data the transfer model would handle, in addition to the various factor score change components during practice on a task $\underline{x}$ noted above, a second task $\underline{y}$ could be given to both the experimental and control groups where the experimental group receives practice on task $\underline{x}$ and the control group does not. The task structure for task $\underline{x}$ and task $\underline{y}$ may be previously determined using other Ss which are matched to the experimental and control groups according to the relevant ability and cognitive style factor scores since there is evidence that different cognitive strategies may alter the factorial composition of a task (French, 1965; Frederiksen, 1969). Given the appropriate task structure of task $\underline{x}$ and task $\underline{y}$ for matched experimental and control groups, it is then a simple matter to obtain difference scores for these groups for performance on task $\underline{y}$. These difference scores for $\underline{N}$ experimental Ss may then be substituted into [12] and different ϕ functions tried out to test the adequacy of the transfer model which takes into consideration both changes in factor scores and factor loadings.

It is possible to conceptualize Fleishman's paradigm in terms of the above account of transfer. Since each trial in Fleishman's conception of a task calls into play a different task structure ($\underline{b}$'s), a given trial may be defined by an equation similar to [1]. Each of Fleishman's trials then become micro tasks and thus the change and transfer model developed here makes Fleishman's one task design really a special case of a multi-task transfer design. In this way both changes in factor loadings and factor scores becomes conceptually possible. The difficulty that immediately presents itself however is obtaining estimates of changes in factor scores during the ongoing macro task or in present terms the sequential micro tasks. Interrupting the task at the end of each trial to obtain estimates of changes in factor scores may be a defensible procedure in a particular case although the dangers of altering the nature of the task should be kept in mind. The advantages however of placing Fleishman's work within the present framework is to call attention to what may be called the more profitable weak disjunctive (either A or B or both) as opposed to the strong disjunctive (either A or B but not both A and B) as paradigms for reconciliating differences. Thus it is conceptually possible that both factor scores and factor weights are changing during practice on a task.

CHAPTER 4

A CONCEPTUAL FRAMEWORK FOR LEARNING EFFECTING THE DEVELOPMENT OF ABILITY FACTORS[6]

Having clarified the use of learning as it is related to changes in ability factors and factor loadings, the next step in this analysis will be to give an example of how one might relate substantive learning principles and changes in ability factors. Gagné's (1965) model is considered as a tentative way of achieving this conceptual linkage, but it should be realized that future work may profit by considering other learning models. In any case, the factor change model outlined above will now be integrated with learning and developmental concepts. Table 2 sets out, in slightly modified form, Gagné's (1965) types of learning. In addition, a taxonomy of some of the learning principles that have received most attention in traditional experimental learning theory are presented showing their hypothesized relationship to Gagné's types of learning. It can be seen that learning principles 1-10 are considered as basic, common to all types of learning. Learning principles 11-28 have been placed under the appropriate type of learning where they would be expected to be most relevant. The latter placement should not be construed as being mutually exclusive, since some of the learning principles 11-28 would be active in more than one type of learning (especially in more complex types of learning that occur after its initial placement in Table 2). The basic learning principles 1-10 will now be stated within the framework of learning effecting the development of

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[6] A version of this chapter is to appear in Buss (1973c).

Table 2

Taxonomy of Learning Principles and Their Relationship to Gagne's Types of Learning

	PRINCIPLES OF LEARNING	GAGNE'S TYPES OF LEARNING						
		S-R	Chaining	Verbal Association	Multiple Discrimination	Concept	Principle	Problem Solving
* 1.	Reinforcement	X	X	X	X	X	X	X
2.	Extinction	X	X	X	X	X	X	X
3.	Stimulus Generalization (Primary, Secondary)	X	X	X	X	X	X	X
4.	Discrimination	X	X	X	X	X	X	X
* 5.	Drives (Primary, Secondary)	X	X	X	X	X	X	X
6.	Yerkes-Dodson Law	X	X	X	X	X	X	X
7.	Massed vs. Distributed Practice	X	X	X	X	X	X	X
8.	Transfer (Positive, Negative, Proactive, Retroactive)	X	X	X	X	X	X	X
9.	Overlearning (Frequency Principle)	X	X	X	X	X	X	X
10.	Readiness (Physiological or Psychological)	X	X	X	X	X	X	X
11.	Sensory Preconditioning (contiguity)	X						
12.	Inhibition (Disinhibition, Spontaneous Recovery)	X						

Table 2 Continued

	GAGNE'S TYPES OF LEARNING						
PRINCIPLES OF LEARNING	S-R	Chaining	Verbal Association	Multiple Discrimination	Concept	Principle	Problem Solving
13. CS - UCS Interval	X						
14. Schedules of Reinforcement (Partial, Continuous)	X	X					
15. Goal Gradient	X	X					
16. Shaping	X	X					
17. Primacy-Recency (Serial Position Effect)		X	X				
18. Higher Order Conditioning			X				
19. Serial Learning			X				
20. Meaningfulness			X		X	X	X
21. Response Generalization				X			
22. Competing Responses				X			
23. Chunking (Clustering)					X	X	
24. Learning Sets					X	X	X
25. Integration					X	X	X
26. Hypothesis Formation					X	X	X
27. Knowledge of Results (Feedback)							X
28. Strategies					X	X	X
29. One trail Learning (Insight)					X	X	X

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*According to Hull-Tolman learning theory, these principles affect performance more directly than learning.

ability factors or changes in ability factor scores. That is, while it is true that much incidental learning may occur without changes in ability factor scores, and ability factor scores are relatively permanent over short periods of time and provide for and set limits as to what may be learned, the present focus is confined only to that situation where changes in ability factor scores over the life-span are effected by learning.

$\underline{P_1}$ A reinforcement is any stimulus event (e.g. confirmation of expectancy) that will increase an ability factor. If reinforcement occurs on a continuous schedule, the maximum possible change in the ability factor score is reached. If, however, the same maximum change in the ability factor score is brought about more slowly by a schedule of partial reinforcement, the effect of the latter will result in greater resistance to extinction.

$\underline{P_2}$ Extinction occurs when there is a decrease of an ability factor score caused by failure of reinforcement. Extinction is more rapid when acquisition occurs under a schedule of continuous reinforcement as opposed to a schedule of partial reinforcement.

$\underline{P_3}$ The involvement of an ability factor under stimulus conditions somewhat different from original learning is called stimulus generalization.

$\underline{P_4}$ Discrimination is achieved when an ability factor is operative in one stimulus situation but not in another.

$\underline{P_5}$ Drives provide the impetus for action as well as defining the direction of behavior (goals). In human ability learning, perhaps the most important drive or motive is achieving cognitive proficiency which facilitates adaptation to the environment.

$\underline{P}_6$ The score of an ability factor is enhanced by an intermediate drive level (inverted U function). However, the more complex the task, the lower is the optimal drive level (*Yerkes-Dodson Law*).

$\underline{P}_7$ Increases in the score of an ability factor is facilitated more by distributed practice than by *massed practice*.

$\underline{P}_8$ *Transfer* occurs when practice on task *x* has an effect on performance on task *y* (positive vs. negative transfer). Transfer effects have their basis, in part, in changes in underlying factor scores brought about by practice on task *x*.

$\underline{P}_9$ *Overlearning* enhances the stability of the level of an ability factor since skills can be evoked at a low threshold.

$\underline{P}_{10}$ Further growth of an ability factor is moderated in the organism by a physiological and psychological *readiness* variable, by which is meant that the individual must be ready for the conditions of the task in the sense that appropriate behavior is in the individual's repertoire.

The above ten basic learning principles are seen as affecting both trait profile and trait pattern. It should be noted, however, that it is individual differences in each of these principles which account for changes in interfactor relationships. Further comment on Gagné's types of learning as they relate to changes in ability factor socres and loadings will be dealt with subsequently. At this juncture, however, I wish to elaborate on how learning principles 1-10 are hypothesized to be involved in the development of ability factors.

The human organism is engaged in a process of adaptive behavior[7] in order to meet the demands of the environment. Control over situational presses leads to the development of cognitive structures (increases in ability factor scores) through learning. With respect to ability factor development, the major drive, motive, energy source, and goal, revolves around mastery of the environment. Growth of abilities lead to achieving this goal. Behavior that leads to successful and proficient environment control or confirms one's expectancies is enhanced and leads to the reinforcement of the hypothesized constructs (ability factors) giving rise to such behavior. If reinforcement is delivered on a continuous schedule, ability factor score changes are more rapid than if a partial reinforcement schedule were in effect. The latter, however, would result in more permanent change, i.e., be more resistant to extinction effects. Inappropriate or mal-adaptive behavior becomes extinguished and facilitates the formation and operation of relevant ability factors. Through the dual process of stimulus generalization and discrimination, the organism learns the generality and specificity of appropriate ability factors. The more frequently an organism puts into operation a given ability factor, the more efficient it becomes, thus solidifying it into a functional unity that has a low threshold for implementation (over-learning). Stimulus generalization can be seen as a special instance of transfer, where competencies and general skills are built up by practice of underlying ability factors which are common to similar tasks. Distributed practice on a given task leads to better performance and thus to greater reinforcement (environmental mastery). As such, it will result in increases in the ability factor that is operative to a greater extent than massed practice, since the latter situation leads to poorer performance, less mastery of the environment, and thus less reinforcement. Throughout this general developmental process, the level that an ability factor can reach is subject to the physiological and psychological readiness of the organism. With respect to psychological readiness, the organism's development is seen as cumulative. Prior learning determines the level later learning can reach.

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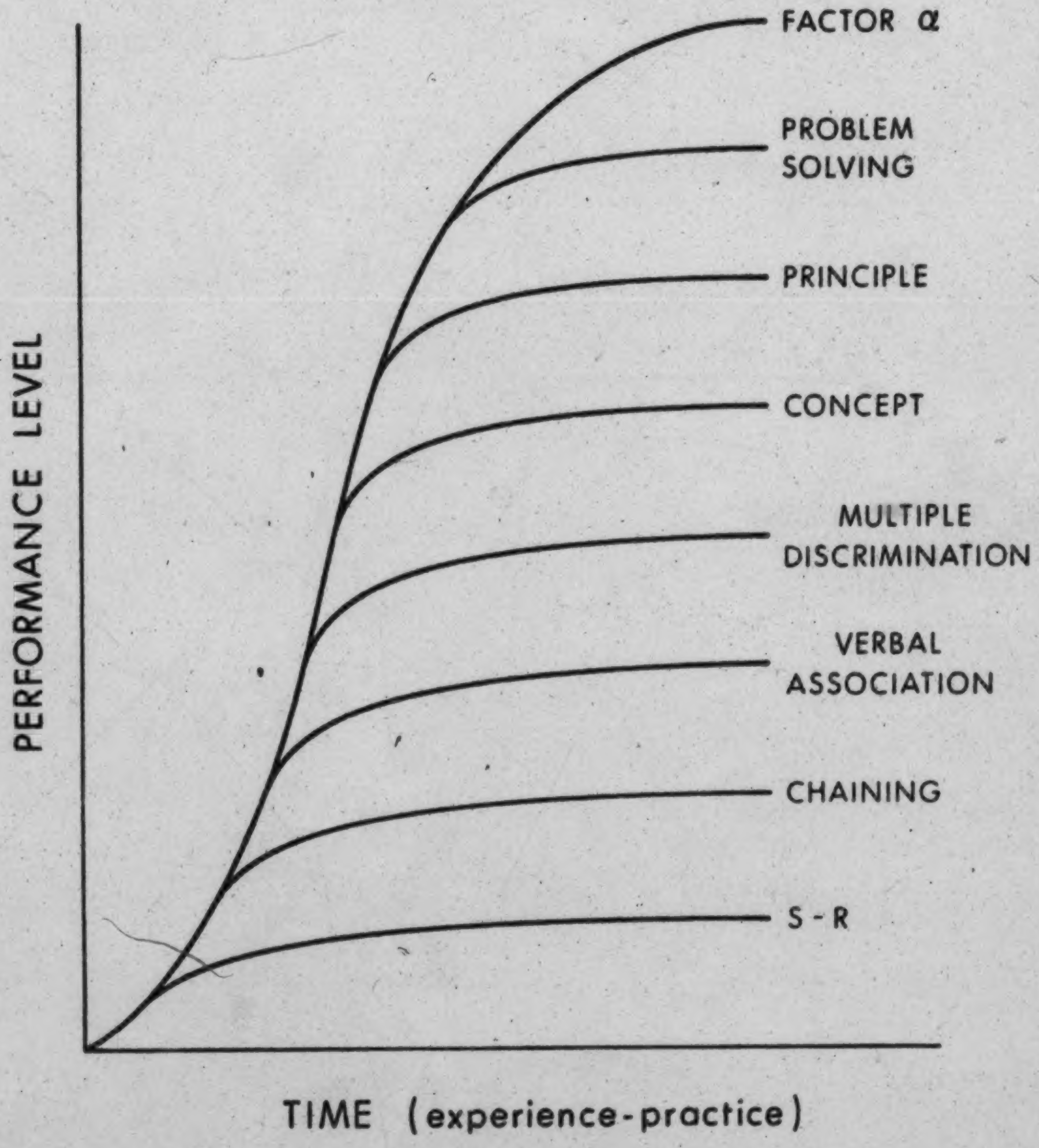
[7] Learning has traditionally been defined as adaptive but the phenomena of latent learning as well as the role of novelty and curiosity have demonstrated that in the short run, nonadaptive learning may occur. If one confines his attention to "intellectual learning" at the human level, then learning may be considered adaptive in the long run.

Learning-Stages, Factor Scores, and Factor Loadings

In Figure 5 the hypothesized relationship between Gagné's types of learning and the development of an ability factor α is represented. In this model, the level of a factor is considered as dependent upon the learning-stage reached in development. The theoretical growth curve of a factor is represented as S-shaped, consistent with the evidence from Thurstone (1955) who applied his absolute scaling methods to his seven primary mental ability factors. As can be seen, later complex learning, upon which a high performance level of factor α depends, is a direct outgrowth of more simple types of learning. Transfer takes place within each of Gagné's types of learning as practice on diverse tasks occurs (learning principle $\underline{P}_8$). In addition, transfer occurs between lower types of learning to higher types of learning (learning principles $\underline{P}_9$ and $\underline{P}_{10}$). Gagné (1965) refers to each of these types of transfer as lateral and vertical respectively. Lateral transfer is left unexplained by his theory, while vertical transfer is

Figure 5

Growth curve of ability factor α according to learning-stage theory. Learning is cumulative and from simple to more complex. The level achieved on each type of learning sets limits on later types of learning and thus the ultimate level a given factor reaches.


FACTOR α
PROBLEM SOLVING
PRINCIPLE
CONCEPT
MULTIPLE DISCRIMINATION
VERBAL ASSOCIATION
CHAINING
S - R
PERFORMANCE LEVEL
TIME (experience-practice)

simply accounted for in that learning at a higher level will be made easier when low level learning has been mastered. Figure 5 has certain implications which go beyond this simple statement. It implies that deficits in lower types of learning have more serious consequences on the ultimate performance level of an ability factor than do deficits in later types of learning. This is indicated in Figure 5 by: low level types of learning branch off the factor performance curve when the latter is positively accelerating whereas high level types of learning branch off when the factor performance curve is negatively accelerating. Therefore, deficits in low level types of learning will have more serious effects on the ultimate factor performance level than will deficits in high level types of learning. Such a model is consistent with Hebb's (1949) theory of the importance of early experience for later functioning, as well as Schneirla's (1957) view that gains from experience are likely to be greatest at early stages of ontogeny. That this same relationship holds for negative influences as well, i.e., brain damage, has also been argued by Hebb (1949), although the evidence here is not conclusive (see Hayes, 1962, for a review).

Evidence for the upper learning stages of Figure 5 comes from Gagné, Mayor, Garstens, and Paradise (1962), although this research was not carried out within the framework of ability factors. More specifically what these researchers did was to examine the learning of mathematics, and found that generally only learners that had mastered capabilities lower in the learning hierarchy were able to master capabilities higher in the learning hierarchy. If one were to consider a more general capability like the deduction ability factor for example, it does not seem unreasonable to hypothesize that growth on this ability occurs as a function of cumulative learning in the manner set out by Figure 5.

Some comment is in order as to the generality of the model as represented in Figure 5. It should be stated that this view of the development of abilities will probably be able to accommodate most of the factors that have received attention, with the added provision that all factors needn't progress through all seven learning-stages. In considering primary ability factors that are considered "established" (see Ahmavaara, 1957; French, Ekstrom, and Price, 1963; Horn, 1972; Pawlik, 1966; Royce, 1973; Thurstone and Thurstone, 1941; for a discussion of their invariance and/or psychological interpretation), some of those that would probably progress through all seven of the learning stages include: verbal comprehension, induction, expressional fluency, number, deduction, spatial relations, syllogistic reasoning, and originality. Taking verbal comprehension as an illustrative example, the process might go something like this: the child learns to attach certain linguistic labels to specific concrete objects (S-R); certain words become connected because of the speaking habits of the linguistic community (chaining); as words acquire richer meaning, associations are built up (verbal association); as similarities and differences between the meanings of words are learned, finer linguistic discriminations may be made (multiple discrimination); abstractions emerge which allow for transcending the immediate situation (concept); concepts are combined, separated, transformed, etc., on the basis of various rules (principle); finally, solutions to problems proposed linguistically are solved on the basis of strategies, which involve the integration and execution of rules (problem solving). It can be noted that idiosyncracies in the development of say, concepts, are possible. However such individual differences may be largely in the connotative meaning as opposed to the denotative meaning. For example the concepts "mother" and "father" will have differing connotations depending upon family environment, although denotatively these concepts will have relatively invariant meanings across familiy environments.

Ability factors that would probably not go through all seven learning-stages include perceptual speed, word fluency, memory, and spatial relations. Thus, perceptual speed would probably progress to the learning-stage of multiple discrimination, consistent with Gibson's (1969) more general theory of perceptual learning, in that perceptual development involves greater and greater differentiation or an increase in the specificity of response to invariant stimulus inputs (i.e., a sharpening of percepts). Word fluency would pass to at least the concept learning-stage, since a rich repertoire of concepts would facilitate such performance. In considering the memory factor, since it involves the retrieval of information which is stored in coded form, a case could be made that it progresses to the concept learning-stage. Short-term memory would probably not fit into this scheme, since it is viewed by some (e.g. Horn, 1968, 1970; Miller, 1956) as basic and unlearned, where the performance level sets a limit on the capacity for information processing. Spatial relations would not pass through a verbal association stage. These examples of ability factors as they relate to the various learning-stages are illustrative only and are not meant to be an exhuastive and definitive statement.

Bracketing for the moment the inter-factor differences in the learning-stage reached, consider now a class of ability factors that are identical in terms of the learning-stages they progress through (it will be easier, though not necessary, to consider those that progress through the full seven stages). One can view the various developmental curves in Figure 5 as ideal types, reflecting the level reached by an ability factor, given that cumulative learning has been maximal up to that particular curve. Of course in human development, the cumulative learning process would not become fixated before reaching the problem solving level for the majority of ability factors. The above model, however, does provide a conceptual framework for theoretically charting the effects of various types of learning in the development of ability factors, and would be especially potent for conceptualizing this process phylogenetically, where certain organisms reach limited levels on ability factors since they do not reach the higher level learning types. Another implication of the various types of learning growth curves is that the transfer asymptote for performance level is reached progressively later in time as one moves from low level to high level types of learning (learning principle $\underline{P}_{10}$). This aspect of the model may also aid in interpreting the development of ability factors phylogenetically, since lower organisms reach full development relatively quicker than do higher organisms. By transfer asymptote, it is meant the level required on one type of learning as a result of practice on those types of tasks, before transfer to the next higher type of learning can occur involving tasks within this higher type of learning. Thus, it is important not to conceive of the learning curve components as reflecting absolute performance over time, but rather as "cumulative, ~~developmental~~, transfer, asymptote, inflection, stage points." To be sure, the human organism continues storing up new S-R connections over the life-span, but in the development of an ability factor, once the organism reaches a critical performance level on S-R learning, he is able to advance to the next type of learning. As the organism progresses to more advanced stages of learning, his ability factor score increases accordingly.

From these considerations, it can be seen how the small gradual changes of the factor growth curve are completely compatible with a qualitative or structural change _model_, since I am adopting the view that Gagné's higher types of learning are an outgrowth of, _but not reducible to_, lower types of learning. This position is also consistent with Table 2 and learning principles 11-28, where it was stated that certain learning principles would be most relevant in certain types of learning. The implication here is that as one ascends the Gagné types of learning from simple to more complex, new forms of learning evolve which are not reducible to lower forms. Such a view is quite consistent with the notion of incremental cumulative learning.

The entire learning process involved in the growht of an ability factor as illustrated in Figure 5 may be conceptualized as hierarchical in nature. The level of an individual's factor score is a direct consequence of how high up the learning type hierarchy he has progressed. Hierarchical models for intellectual growth with respect to learning have been advanced by Gagné (1968a, 1968b), but they have not until now been put into the framework of ability factors. Hierarchical conceptions of ability factors exist (Burt, 1949; Cattell, 1971; Vernon, 1950), but they have been devised in accordance with mapping out the existent nondevelopmental structure of the entire ability domain.

The present account of the relationship of learning to the development of an individual's ability factor scores is quite analogous with a stage view of cognitive development (e.g. Bruner, 1964; Piaget, 1970). The stage concept has been discussed at some length (Flavell, 1963, 1971; Flavell and Wohlwill, 1969; Hunt, 1969; Inhelder, 1956; Kessen, 1962; Pinard and Laurendeau, 1969; Reese and Overton, 1970). The essential properties of stages are that they comprise an invariant sequence in which later stages subsume earlier ones and thus provide for integration, and that a given stage is defined by structural wholes rather than isolated behavioral components. In terms of the present model, Gagné's types of learning may be viewed as an invariant sequence of learning-stages, where the structural wholes defining each stage consist of the different kinds of learning and principles as set out in Table 1. Empirical evidence supporting this view is reviewed by Gagné (1968b), although he does not use the word "stage". Quantitative changes in factor scores would occur within any learning-stage, and qualitative or structural changes would parallel the transformations from stage to stage. Once the individual reaches the terminal learning-stage for a given factor, further changes in factor scores would be quantitative in nature, which is consistent with the views of Flavell (1970) and Kohlberg and Kramer (1969) as to the kinds of changes in the adult's cognitive structure. It would be expected that the majority of ability factors would progress simultaneously through the various learning stages, although possible exceptions would resemble Piaget's concept of horizontal décalage.

That factors may undergo structural transformations over the life-span has been noted in Chapter 1. This orientation towards considering dimensions of individual differences through the life-span may be contrasted with that approach which attempts to demonstrate the stability of factor pattern matrices from occasion to occasion through the use of factor matching procedures (e.g. Meredith, 1964). Undoubtedly, the concern with the latter issue has been prompted by the belief, that, if factors are to be useful theoretical constructs, their invariance must be established across different subjects, variables, and occasions. It should be noted, however, that these two major concerns are not incompatible, since the factor matching procedures may be used for establishing the boundary conditions for structural stability of factors. It may be hypothesized that these boundary conditions may line up with the different learning-stages. That is to say, in the case of ability factors, the various learning-stages that have been considered may be viewed as providing the theoretical underpinnings for such changes in the salient variable loadings that define the factor. For example, the types of items that would tap, say, the factor spatial relations, may be viewed as intimately linked to the learning-stage of an individual (e.g. an S-R response that a circle and a square are not the same thing vs. a highly abstract geometric problem solving item that involves plane geometry theorems).

Research aimed at testing the model described here would involve essentially two steps. First, it would be necessary to establish age norms for the various learning-stages, and further, for individual ability factors (horizontal déclage). Consistent with Piaget's stage theory of cognitive development, it would not be necessary that such age norms be identical across different cultures, since the critical point here is that they form an invariant sequence. Having done this, the second step would be to carry out a typical training experiment for a particular learning-stage, where the experimental and control groups are compared on the ability factor in question before and after training or nontraining respectively. If, for example, in considering the abiltiy factor spatial relations, the subjects are at the concept stage (i.e., they have a firm graps of such things as a square, circle, triangle, circumference, right-angle, etc.), training for the principle learning-stage would involve such things as the pythagorean theorem, that the sum of the angles in any triangle equals 180 degrees, that the diagonal in a square separates two identical isosceles triangles, etc.

In conclusion, it should be noted that the purpose of the present chapter has been to integrate diverse psychological domains and concepts. Mechanistic learning views of development (learning principles, incremental growth, etc.), organismic developmental concepts (qualitative change, stage, etc.), and dimensions of individual differences (factors), are all brought together under the same umbrella. Such a goal is an important one to the extent that major developmental theorists (e.g. Piaget) are largely not concerned with individual differences (Butcher, 1968; Flavell, 1963; Hunt, 1961; Wohlwill, 1970a; Zigler, 1963). By the same token, there have been those who have called attention to the lack of developmental thinking in the study of individual differences and have attempted to redress this deficiency (e.g. Emmerich, 1968; Horn, 1968; Wohlwill, 1970a). The meshing of mechanistic or learning views of development and Piaget's organismic approach has already been initiated (Berlyne, 1962; Hunt, 1969; Stevenson, 1962). All this is by way of saying that the aim of the present account is to further recent theoretical integrative trends that are certain to increase our understanding of psychological development. In the following chapter, a closer look is given to developmental changes of ability factors over the life-span.

CHAPTER 5

FURTHER ISSUES IN INTEGRATING ABILITY FACTORS AND DEVELOPMENTAL CONCEPTS

Recently there have been attempts to close the conceptual gap between the individual differences approach and developmental concerns (Emmerich, 1968; Wohlwill, 1970). Thus Wohlwill (1970a) mentions two ways of bringing the study of individual differences into a developmental context: differences in the general form of an invariant developmental pattern (parameter variation), and differences in the pattern itself (non-invariant developmental pattern). Emmerich (1968) has contrasted and to a limited extent integrated certain aspects of what he sees as the three major approaches to structural development in personality, i.e., the classical, differential, and ipsative. The classical view subscribes to universal invariant sequences of stages, the differential stresses the assortment of subgroups with respect to differential dimensions in the course of development, and the ipsative considers intra-individual consistencies and change over time.

The purpose of the present chapter is to push further towards the goal of integrating the study of individual differences and developmental concerns. This is mediated by outlining three basic ideal factor relation types i.e., divergence, convergence, and parallelism. Each of these types of relations are applied to the trait pattern (Anastasi, 1970) or factor inter-relationships. Additional concepts are considered which provide for an analytic statement as to the possible ontogenetic changes in structure, i.e., cumulation vs. non-cumulation of factors and higher order factors. The model developed is applied to the classical, differential, and ipsative approaches to developmental change, as well as organismic developmental concepts. In this way, a beginning is made to integrate multivariate approaches to the structuring of behavioral change and mainstream developmental concepts. This goal is an important one to the extent that multivariate change models must be advanced enough to be able to respond to the major issues which have dominated the field of developmental psychology. Multivariate approaches to the structuring of change must examine these issues within its own framework and develop models that are sensitive to the conceptual demands of this area.

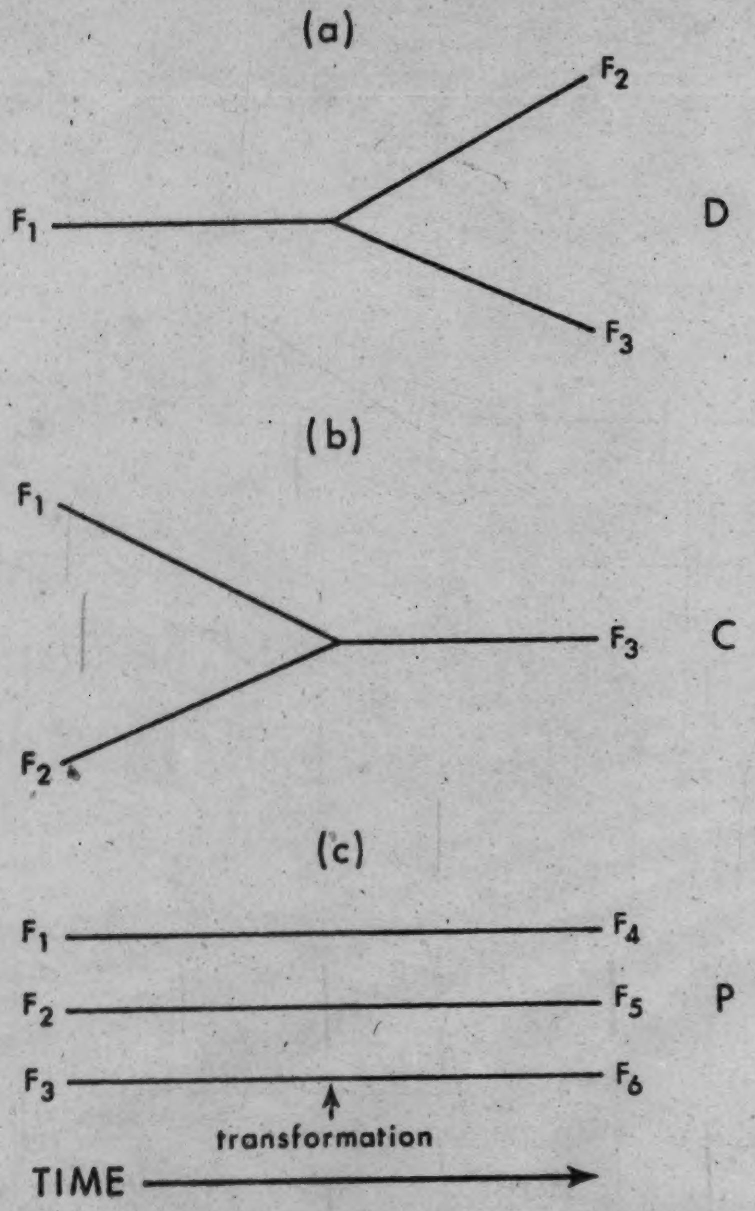
Underlying Structure and Changes in Factors

Trait Pattern

Figure 6 (a), (b), and (c) illustrate in a simplified manner the three basic ideal factor relation types, i.e., divergence (D), convergence (C), and parallelism (P) respectively. These ideal types of inter-factor relations may be considered mutually exclusive and jointly exhaustive, since, as will be demonstrated below, any structure of factor relations may be decomposed into varying combinations of these basic relation types. Several comments can be made concerning the factor relations expressed in Figure 6. Each of these basic types are distinct polyadic relations, since more than one factor is required to define each type and must be considered with respect to each other. Divergence may be considered the inverse of convergence, since they are identical in form although reversed in direction with respect to time. The non-parallel lines in Figure 6 indicate changing covariance between factors, while parallel lines reflect a constant covariance (not necessarily zero)

Figure 6

The three basic ideal factor relation types of divergence (D), convergence (C), and parallelism (P).


(a)
F1
F2
F3
D
(b)
F1
F2
F3
C
(c)
F1
F2
F3
F4
F5
F6
P
transformation
TIME

across time.[8] The transformation point indicates a major change in the salient variable loadings that define a factor such that a different factor or factors occur after such critical events. It should be noted that the logic of the D and C cases require that a transformation actually occur, while this is not necessary for the P case. That is to say, it is possible in the P case, where the factors F_4, F_5, and F_6 are replaced with factors F_1, F_2, and F_3 respectively, that no transformation takes place. This special case of P may be called nontransformational parallelism, and is interpreted as constant inter-factor covariances across time between factors whose meaning remains unaltered (constant salient variable loadings). Unless otherwise indicated, P will stand for transformational parallelism below. The three basic ideal factor relation types may seem rather simple prima facia. As demonstrated below, however, their possible permutations plus additional growth properties to be considered, allow for considerable power for potentially teasing out the complexities of age-related changes of ability factors.

A similar conception to the present one has been advanced by Van Den Daele (1969), although he was concerned exclusively with qualitative models where the unit of analysis was stages rather than factors. Van Den Daele's paper is an important contribution to the analysis of qualitative stage models, and is the impetus for parts of the present section insofar as some of the distinctions that he has made are applied and extended within a multivariate framework. Van Den Daele (1969) provided for two cases of mixed multiple progression types, or what has been called here basic ideal factor relation types. These were partially convergent yet overall divergent, and partially divergent yet overall convergent. Focusing now on factors and allowing the overall factor relation type to be symbolized by its upper case letter and the partial factor relation type to be symbolized by its lower case letter, these two mixed factor relation types may be represented by Dc and Cd respectively (Figure 7 (a) and (b)). In Figure 7, the various factors and transformation points have not been labelled in order to facilitate ease of presentation. At each transformation point, however, changes in salient variable loadings or structural change occurs, which results in new factors.

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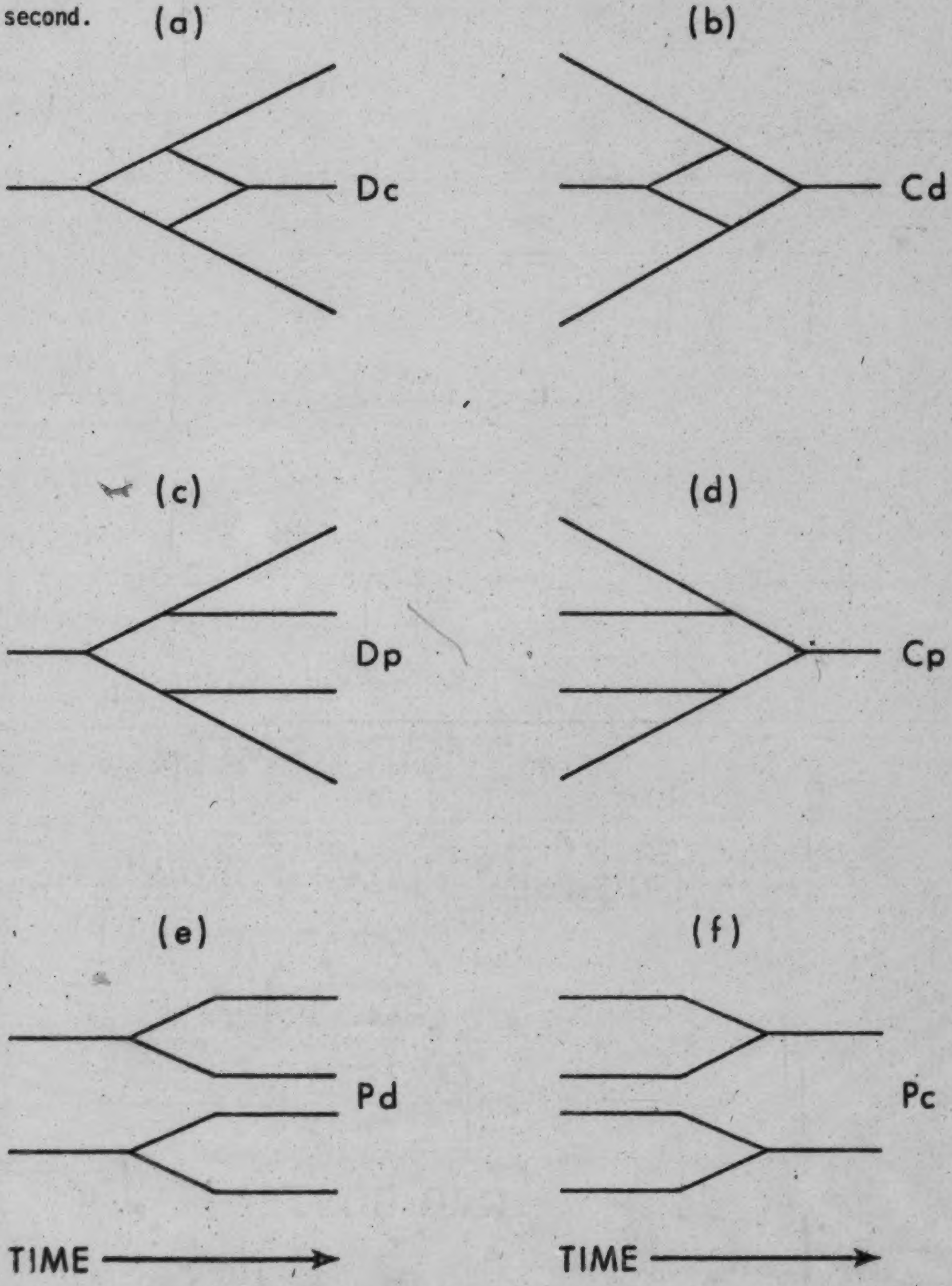
[8] For a discussion of identifying changes in factor covariances, see Mulaik (1972, p. 358).

As Figure 7 reveals, there are a total of six simple mixed types of ordered combinations, where by simple we mean selecting and arranging two factor relation types from a total of three possible without replacement or duplication, i.e., 3!/(3-2)!. This principle may be extended to $\underline{n}$ selection or reference points through time while allowing for duplication. The number of possible mixed factor relation types then becomes $3^{\underline{n}}$. For example, if $\underline{n} = 6$, then there would be 729 possible mixed factor relation types! The assumption that the first basic factor relation type in a series would be over-riding, while the remaining basic factor relation types would be partial or subsumed, need not be made. Indeed, as $\underline{n}$ increases, it is improbable (though possible) that there would be a single basic factor relation type modulating the remainder of a particular sequence. More probable would be a number of major (i.e. over-riding) factor relation types, interspersed with minor (i.e. partial) factor relation types.

Figure 7

The six possible simple mixed factor relation types, where only two basic ideal factor relation types are considered and the first overrides the second.

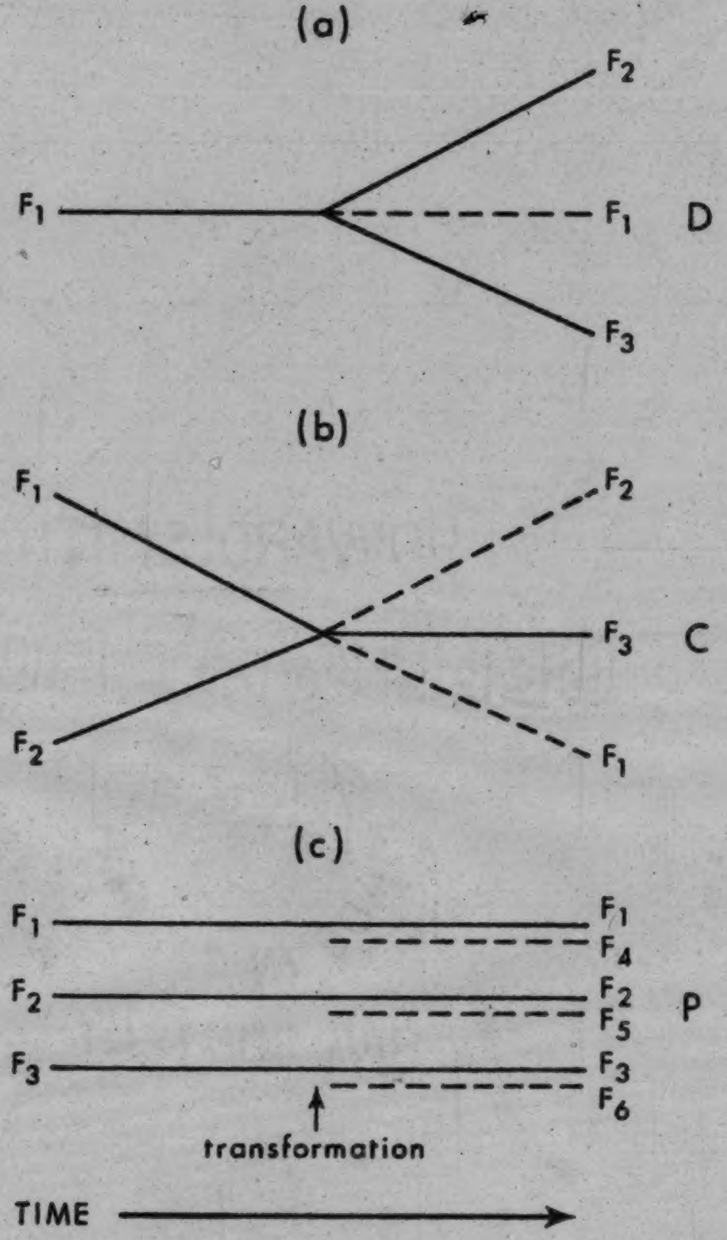

(a)
(b)
Dc
Cd
(c)
(d)
Dp
Cp
(e)
(f)
Pd
Pc
TIME
TIME

One property of the basic ideal factor relation types which is orthogonal to the distinctions made thus far, is that the changing ontogenetic factor relational structure may or may not be cumulative. By this it is meant that in going from one factor relation type to the adjacent one in a particular sequence, there may or may not be a cumulation of pre-existing factors. This point may be appreciated by reference to Figures 6 and 8, since they are identical except for the nonpresence and presence of cumulation respectively. Thus in Figure 6, the transformations which the pre-transformation factors undergo result in new factors which take the place of pre-existing factor forms. In Figure 8, the pre-transformation factors are retained as well as the new factor forms post the transformation point. In other words, factors cumulate, since earlier factors remain in the repertoire after critical transformation periods. This process produces a paradoxical result in Figure 8 (b), where the cumulative convergent case actually yields more factors after the transformation point.

The analysis presented above may be thought of as being more systematic than that presented by Coan (1966). Coan's convergence and divergence remain in the present account, but it was argued that the third pattern of parallelism is needed to complete the number of factor relation types. It was also noted that D, C, and P, are all of the same logical type, i.e., express polyadic factor relations, and that the inter-relationship between D and C reveals the interesting logical property that one is the inverse of the other. Coan's list of five types of ontogenetic change in factors (see Chapter 2) are not formally grouped and contrasted according to their logical properties. By simply listing five constructs there may be a tendency to inappropriately

Figure 8

The three basic ideal factor relation types of divergence (D), convergence (C), and parallelism (P) where the property of cumulation is in evidence after the transformation point.


(a)
F1
F2
F1
F3
D
(b)
F1
F2
F2
F3
F1
C
(c)
F1
F1
F4
F2
F2
F5
F3
F3
F6
P
transformation
TIME

consider them to be all of the same logical type. The point to be made here is that Coan's factor metamorphosis, emergence, and component interchange form a group of change properties that may apply to any of the basic factor relation types D, C, and P where cumulation or non-cumulation is in effect.

Recent reviews of the empirical evidence for the ability factor differentiation hypothesis (Anastasi, 1970; Baltes and Nesselroade, 1973; Reinert, 1970) indicate that over the life-span, an integration-differentiation-integration sequence seems most likely and corresponds to early childhood, childhood and adolescence, and adulthood and old age respectively. In terms of the present model, this would be represented as a CDC sequence. Recently, McCall, Hogarty, and Hurlburt (1972) have extensively examined developmental transitions in infant or sensorimotor intelligence. What they did was to correlate factors at the ages of 6, 12, 18 and 24 months. Correlation matrices depicting the factor inter-relationships graphically reveal a complexity in which convergent, divergent, and parallel components are present. Since the authors comment that "the network of transitions between skills at one age and another is likely more specific and complex than once thought" (p. 746), and since they were focused on only an 18 month time span, it can be appreciated that the model developed here may prove quite useful for abstracting some degree of order in the changing pattern of ability factors over the life-span.

It is also possible to incorporate in the present scheme Guilford's (1967) view of trait patterns through the life-span. Guilford's adoption of orthogonal rotational procedures has forced him into a position where he is unable to accept any type of structural changes in the factor inter-relationships across the life-span. Guilford's strategy has been to map out the existent ability structure, and attempts by others (e.g. Burt, 1949; Spearman, 1927; Thurstone, 1938; Vernon, 1950) have been carried out in a similar vein. However, it is a misguided hope to establish a single structure of the abilities domain that will hold across all ages, since this would contradict the major developmental principle that structures become increasingly differentiated with age while simultaneously becoming hierarchically organized (see Chapter 6). Recently, there have been those (Reinert, 1970; Baltes and Nesselroade, 1973b) who have noted that certain models of ability structure are more appropriate at certain ages, e.g., Spearman's (1927) model at early stages and Thurstone's (1938) and Guilford's (1967) at later stages. Returning, however, to Guilford's view within the present context, it may be considered as a special case of P without cumulation. The rationale for considering it as such is that P requires that the factor inter-relationships remain invariant across time, regardless of whether they are oblique or orthogonal.

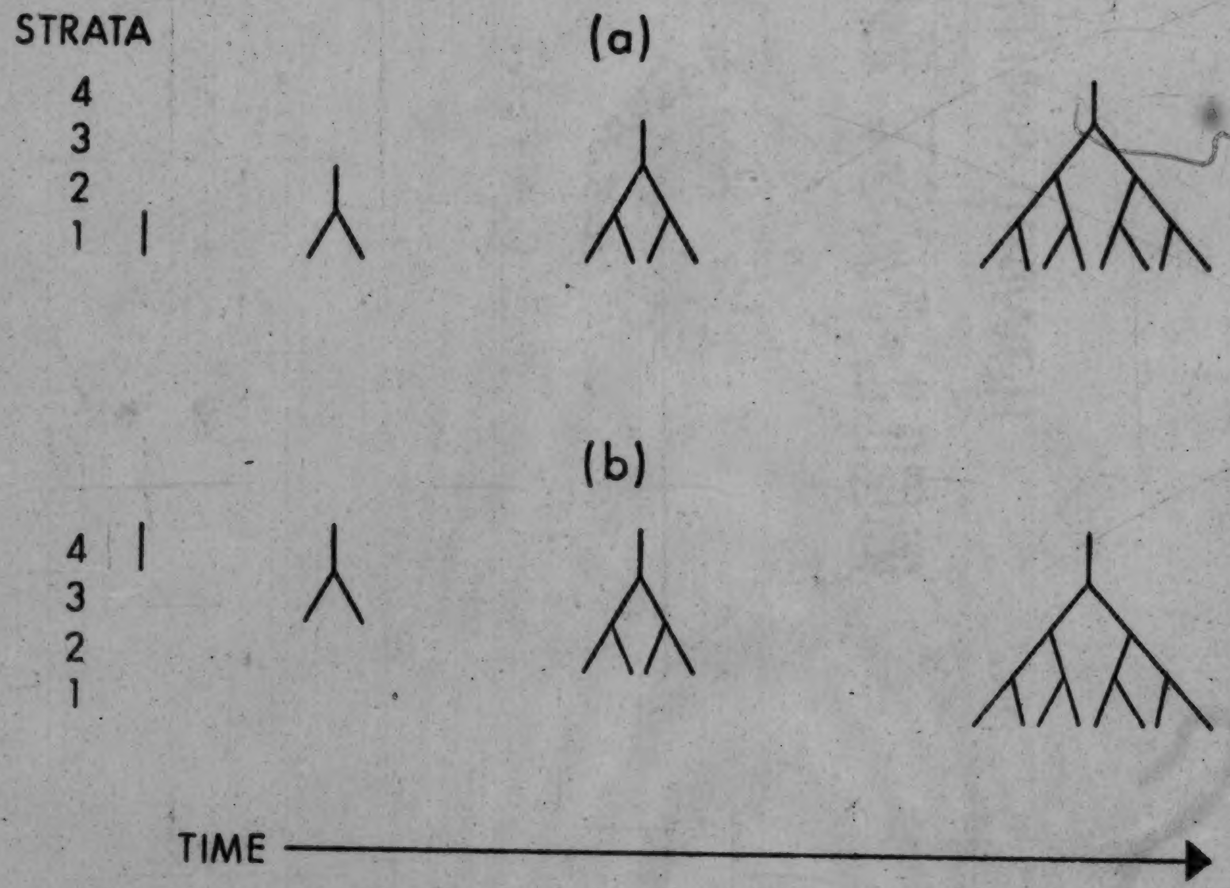
Higher-Order Factors

The question of higher order factors is quite separate from the discussion of the basic ideal factor relation types. Thus, in the D type factor relation, the pretransformation factor may or may not be a higher order factor. Similarly, the post transformation factor in the C type factor relation may or may not be a higher order factor.

Figure 9 (a) illustrates the simple case of a recurrent divergent sequence in which there is a progression from lower order to higher order factors. Higher order factors increasingly emerge at increasingly higher levels during this process of ontogenetic change.

Figure 9

The two possible types of ontogenetic processes for higher order factors where the only difference is the relationship between factor strata and time.


STRATA
(a)
4
3
2
1
(b)
4
3
2
1
TIME

This recurrent divergent view of the ontogeny of higher order factors does not have a monopoly in accounting for the appearance of higher order factors. Figure 9 (b) illustrates another possibility, where the only difference is the relationship of order to the divergent process. In this view, the higher order factor forms are present at the beginning of the ontogenetic change process, and later emergence of higher order factors are progressively at a lower order. Strictly speaking, rather than higher order factors, we are concerned in Figure 9 with higher strata factors. The distinction between order and strata has been made by Cattell (1965), in that the former refers to the operational results of a single experiment while the latter refers to the "real" orders among factors. Thus in Figure 9 (b), the first series of factor groupings reflect fourth, third, etc. stratas, even though there are only one, two, etc. levels being considered. These two views, of course, represent idealizations for the purpose of attaining conceptual clarity as to the logical possibilities of the ontogeny of higher order factors. They are by no means incompatible within a given ontogenetic pattern and may, in fact, coexist in different degrees. It should be noted that although Figure 9 represents nonoverlapping hierarchies where factors are neatly subdivided, an overlapping model is more probable where a lower order factor is linked to two or more higher order factors (Horn, 1972). The latter view is actually embodied in the basic factor equation as applied to higher order factors (see below).

Up until now, the ontogenetic structuring of higher order factors has been exclusively confined to the divergent view as expressed by Figure 9 (*b*). Cattell (1971) and Horn (1966b) hold the view that, in obtaining higher order factors, one is structuring increasingly broad constructs which are of a greater historical vintage. The present analysis will hopefully provide the conceptual framework for dealing with the complexities of this question of the ontogeny of higher order factors and build upon the pioneering work of Cattell and Horn.

It will be useful to look at the two views of the ontogeny of higher order factors within the context of the basic factor equation with the variance associated with uniqueness set aside, i.e.,

$$F_{fi} = b_{f\alpha}F_{\alpha i} + b_{f\beta}F_{\beta i} + \cdots b_{fk}F_{ki} \qquad [13]$$

where F_{fi} is the factor score of individual i on the primary factor f, the b's are the higher order factor loadings, and the F's on the right side of the equation are higher order factor scores. Equation [13] as it is stated reveals some interesting observations once the two types of ontogenetic processes for higher order factors are superimposed upon it. Considering [13] as an expression of the higher order factor development in Figure 9 (*b*), the interpretation would be that the higher order factors account for the lower order factors. In fact, a causal interpretation in terms of the higher order factors "causing" the lower order factors is defensible here, since the ontogeny of any higher order factor, as revealed in Figure 9 (*b*), is always at an earlier time with respect to the factors at the immediate lower level.[9]

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[9] Of course one can never "prove" causality within the present model, but a particular causal theory can be consistent with the factor model. Experimental manipulation is an obvious ingredient that is lacking here and which would provide stronger grounds for inferring causality.

In considering the process indicated by Figure 9 (a), a radically different theoretical interpretation of the model expressed by [13] is necessary. In this case, the ontogeny of any higher order factor always follows that of the factors at the immediate lower level. Given this time sequence between lower and higher order factors, equation [13] now becomes a statement of the ontogenetic decomposition of a lower order factor into its constituent higher order factor components, and if there is any cuasal statement to be made it must now be in the direction of from the left to right side of [13].

The model represented by Figure 9 can be further subdivided into both cumulative and noncumulative types. In considering the last structure in Figure 9 (b) for example, the cumulative case would be a decomposition of a higher order factor in terms of which lower order factors it now subsumes and to what extent (the b's), while the noncumulative case would be a decomposition in terms of which lower order factors and to what extent (the b's) the higher order factor has now become. In the first instance, the higher order factor would have a continuing existence, whereas in the second case, its existence would have been terminated at the transformation point (compare Figures 6 (a) and 8 (a) assuming F_1 is a higher order factor). All this discussion is by way of stressing the complexity of explaining the ontogeny of factors in terms of other factors, and points out the need to be aware of the various ontogenetic processes of higher order factors, since this will determine the theoretical interpretation of the model represented by [13]. It appears that Cattell (1971), for example, views higher order factors in both the cumulative and noncumulative sense as represented by Figure 9 (b) (higher order factors precede and determine lower order factors). An example of the former would be the higher order ability factors fluid and crystallized intelligence, while that of the latter would the concept "historical fluid intelligence" (Cattell, 1971, p. 119).

Implications

Classical, Differential, and Ipsative Approaches

It is now possible to indicate to what extent the model outlined here integrates the three main approaches to developmental questions that Emmerich (1968) has considered, i.e., the classical, differential, and ipsative strategies. These three developmental approaches may be recast in the present treatment by asking to what extent does the model elaborated here provide for detecting general ontogenetic change forms, interindividual differences in ontogenetic change, and intraindividual ontogenetic changes, where the unit for analysis is factors. The merging of these three concerns into an integrated view of ontogenetic change has yet to be done, although there have been those (Baltes and Nesselroade, 1973; Wohlwill, 1970a) who have gone so far as to suggest that at the very core of studying developmental change must be an approach concerned with intraindividual change and interindividual differences in intraindividual change.

General ontogenetic change forms require an invariant sequential pattern that has some degree of generality across individuals. In the present context, this would mean the establishment of an invariant sequential pattern of such things as the basic ideal factor relations and their higher order components, as well as the developmental pattern of higher order factors. Of course, it is not necessary to require that such general ontogenetic change forms _must_ occur for all individuals, but rather that if a given structure is in evidence for a given domain, culture, population, etc., then there is a prescribed sequential invariant pattern which must have preceeded this structure and which has the characteristic of inevitability or even logically necessity with respect to such a structure. Thus the description of general ontogenetic change forms becomes intimately interlocked with the developmental theory of such change. Such theory (e.g. Piaget, 1970, and as interpreted by Flavell, 1963) is almost completely lacking within the framework of multivariate ontogenetic change, although a beginning has been made in this direction in Chapter 3, where the concept of invariant learning-stages was tied to both changes in factor scores and factor loadings. Perhaps the lack of sophisticated descriptive models for abstracting the latent structure of the ontogeny of factors has contributed to the lack of developmental theory in this area, since adequate description of a phenomena must preceed explanation, although these two concerns are not always distinct and often merge together. Hopefully, the model that has been outlined here will go some way in providing the means for an adequate description of age-related factor change, thus preparing the way for some multivariate developmental theory.

Assuming the establishment of general ontogenetic behavioral change forms, the very fact that the unit of analysis in the model being considered is factors, allows for interindividual differences in ontogenetic change. Factors are defined and determined upon the very notion of variability. Thus, although individuals may be subject to the same general ontogenetic change form, interindividual differences in ontogenetic change may be captured by the interindividual differences on the factors which define the general ontogenetic form.

Another possible route to interindividual differences in ontogenetic change would be that situation where different general ontogenetic change forms are in effect for two groups. This possibility assumes alternate developmental sequences and is quite plausible, considering the effects that differences in population (e.g. culture) may have on developmental phenomena. Also, alternate developmental sequences are much more probable in certain domains than in others (Van Den Daele, 1969). These two types of interindividual differences in ontogenetic change readily lend themselves to intraindividual ontogenetic change considerations. Thus, in considering the case where a single general ontogenetic sequence of basic ideal factor relation types is in effect, both interindividual differences and intraindividual changes may be specified in terms of factor scores. For the case where there is alternate general ontogenetic sequences for two specific groups defined by, say, cultural differences, this would increase the interindividual differences across groups though, of course, not necessarily the within group interindividual differences or intraindividual differences.

The three types of ontogenetic changes as formulated above within the framework of factors, reveal some interesting differences when compared to traditional views of these approaches. This is especially true of general ontogenetic change forms. In considering the latter, traditionally a general ontogenetic change form may be considered as a property of a group of individuals. In other words, the pattern of change holds for each individual of a particular group. In the present factor framework, a general form or pattern of change (remembering that a pattern is defined here as changes in the interfactor covariances) is a property of the group per se, since the entire group of individuals is necessary in order to define such patterns, yet such a pattern is not a property of a particular individual. The only way it would make sense to speak of a general ontogenetic change form applying individually (and thus simultaneously to a group of individuals) within the framework of factors, is if the changing pattern of factors were determined individually by such methods as P-technique (correlating variables across occasions for one individual) or similar methods as discussed by Coan (1966). What would be changing here would actually be the trait profile or factor scores rather than the trait pattern, given the definition of P-technique. It should also be noted, however, that replication of the cross-individual factors as defined by P-technique is still in its infancy (Luborsky and Mintz, 1972). In terms of intraindividual ontogenetic changes, it should be noted in passing that P-technique is a method par excellence for structuring such change.

In summary then, the change model developed in this chapter may specify general ontogenetic sequence forms and allows for both interindividual differences and intraindividual changes within such forms. Thus, the proper approach to developmental behavioral change is to study interindividual differences in intraindividual changes, where the latter is an expression of general ontogenetic invariant sequences which are not necessarily universal in nature.

Traditional Developmental Concepts

There are several important developmental issues which have yet to receive the attention they deserve from a multivariate perspective.

Concerning the concept of development, there has been several symposiums devoted to exploring its nature (e.g. Harris, 1957; Stevenson, 1966) as well as several individual efforts (Flavell, 1963; Flavell and Wohlwill, 1969; Kaplan, 1967; Reese and Overton, 1970; Zigler, 1963). Restricting our treatment to Piaget's and Werner's views and their interpreters, development is considered by these two giants in the field as an epigenetic as opposed to a preformative process. Influenced by Piaget, Flavell (1963) and Hunt (1961, 1969) have gone to some length to argue for the differentiation and emergence of cognitive structures that are not reducible to previous structures (epigenesis). These structures gradually come about through an active organism interacting with his environment. Hunt (1961) amasses considerable evidence against the doctrine of biological preformation (disproved by Wolff in 1768) and psychological preformation. Given the general acceptance of an interactional, epigenetic view of the development of cognitive structure, it is somewhat embarrassing that there are some (in the logical sense meaning at least one) who categorically deny its validity and cling to the outmoded view of psychological preformation. Thus Guilford (1967) has been forced into this absurd position by the adoption of a certain methodology (orthogonal rotation procedures). Guilford's position regarding the ontogeny of factors (a special case of parallelism) does have its place in the present overall scheme of multivariate ontogenetic change. However, it cannot be considered as the paradigm for psychological development.

Werner's organismic-developmental approach (Kaplan, 1967; Werner, 1957) is characterized by his orthogenetic principle, which states that "Insofar as development occurs in a process under consideration, there is a progression from a state of relative undifferentiation to a state of increasing differentiation and hierarchic integration" (Kaplan, 1967, pp. 82-83, his italics). Piaget has a similar conception of development (Flavell, 1963) insofar as schemes differentiate and then integrate to form more complex superordinate schemes. Related to this is the notion that cognitive structures which have differentiated at earlier stages become integrated or subsumed within more advanced stages. These two co-existing antithetical Hegelian properties of cognitive development, are readily accommodated by Figure 9, which reveals the two ideal processes whereby higher order factor develop over time. In both Figures 9 (a) and 9 (b), there is, respectively, progressively higher or lower levels of differentiation. In Figure 9, the relationship of strata and time provides for increasing differentiation with hierarchic integration.

Both Werner and Piaget's developmental approach is organismic, as opposed to mechanistic (Reese and Overton, 1970). This means, among other things, that the organism is viewed as active rather than passive and constructs reality with the aid of a highly organized set of psychological structures.[10] Questions of change focus on changes in the qualitative nature of psychological structures and changes in the organization of such structures. Such a view is quite compatible with the factor model, since here the organism is viewed as bringing to a situation a highly organized system of structures or factors that are subject to qualitative or structural changes (see Chapter 2), although the factor model does not specify how the organism selects and uses these factors in performance. The entire structure of factors may undergo reorganization through the basic ideal factor relation types. The organismic model also implies a holistic approach, i.e., the organism's behavior is best understood and predicted by a knowledge of its overall psychological structure. In accounting for any given behavior from a multivariate perspective, the basic factor equation makes full use of the overall structure. This point may be more fully appreciated by referring to equation [13] where one may consider more generally a certain behavior or performance as being specified by the factor loadings and factor scores rather than a factor.

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[10] It should be noted that the concept of "structure" within the factor analytic model is actually not as rich as how this term is used within cognitive psychology. The latter describe knowledge structures which consist of nodes connected by relational links which themselves are defined by other nodal connections (Wilson, 1972), within the system, while the structure within the factor model (i.e. vectors of factor loadings and scores) do not have referents that are defined within that system.

The organism's overall psychological structure is considered in this model in several ways. First, the relevant factors called into play may be from several distinct domains, such as the ability, temperament, motivation, etc. Secondly, the factors are not necessarily restricted to those of the same order. Methods exist (Cattell, 1965; Schmid and Leiman, 1957) for including both primary and higher order factors. Thirdly, the unique set of factor scores which exist for each individual may be seen as taking account of an organism's psychological profile or structure. Finally, if there are changes in the interrelationships between the factors (changes in overall structure), as provided for in the oblique case where factors are allowed to become correlated, then there will be appropriate changes in the weights ($\underline{b}$'s) if one factors correlational matrices rather than variance-covariance matrices (see Appendix A).

Consider now an important aspect of Piaget's organismic model of development, i.e., the invariant functions of assimilation and accommodation. Very simply, assimilation refers to that process whereby the organism structures or constructs reality by imposing his existent schemes or psychological structure upon reality. The world is assimilated and interpreted in terms of these stable structures which the organism brings to any situation. Accommodation is the antithesis of assimilation and refers to that process whereby reality imposes itself upon the organism such that his cognitive structures undergo change or accommodate to the demands of reality in order that he might more adequately deal with that reality. This assimilation-accommodation process has a direct counterpoint in the factor model outlined above. In considering assimilation, it would correspond to that period when the ideal basic factor relation type of parallelism without transformations is in effect, since this would indicate a stable structure that may undergo quantitative change, but the essential nature and interrelationships of the factors would remain invariant. When nontransformational parallelism is in effect, the organisms bring to any situation a stable psychological structure which is imposed upon reality, thus providing the means to deal with that reality. Comparable to accommodation would be the ideal basic factor relation types of divergence, convergence, and transformational parallelism. In interacting with the environment, organisms encounter certain demand situations which contribute to the restructuring of inter-factor relationships and qualitative changes as to the nature and number of factors. The three basic ideal factor relation types provide the vehicle for accommodative change of the psychological structure that equips the organism for more adequate dealings with reality. This interpretation (like others in this section) runs the risk of being accused of simplifying very complex theoretical developmental constructs. However, the aim is not to 'water down' such sophisticated developmental conceptualizing, but rather to bolster the conceptual level of multivariate approaches to developmental issues. Once a tolerable level has been achieved in this regard, the power of multivariate approaches to the structuring of behavior in general may be more fruitfully brought to bear to questions of ontogenetic change.

In the following two chapters, various descriptive data gathering strategies are outlined that deal with obtaining change data. In Chapter 6, the developmental models of Baltes (1968), Cattell (1970), and Schaie (1965, 1970, 1973) are critically reviewed and extended in terms of mapping out changes in factor interrelationships. In Chapter 7, a more general framework is advanced for dealing with person X variable X occasion data. The latter is couched within the concepts of inter-individual differences, intra-individual differences, and intra-individual changes.

CHAPTER 6

AN EXTENSION OF DEVELOPMENTAL MODELS THAT SEPARATE ONTOGENETIC CHANGES AND COHORT DIFFERENCES[11]

It has recently been proposed (Baltes, 1968; Cattell, 1970a; Schaie, 1965, 1970, 1973) that traditional longitudinal and cross-sectional methods used for assessing ontogenetic (age-related) changes, are at best incomplete, and that they should be viewed as special cases of a more general developmental model. The inadequacy of the simple cross-sectional method includes, among other things, the confounding of cohort (generational) differences with genuine age-related changes. The deficiencies of the simple longitudinal method include the inability to assess cohort effects, as well as contamination effects brought about through repeated measures on the same individuals. For a more detailed discussion of these and additional criticisms of the two traditional developmental models, the reader is referred to the original articles as well as the references cited therein. It can be noted, however, that a resolution of some of the main criticisms of these two models has involved the adoption of sequential strategies (Baltes, 1968; Schaie, 1965), a term that has not meant the same thing for Baltes and Schaie, and which will become clarified subsequently.

The purpose of the present chapter is to provide a brief and selective critical review of the developmental models of Baltes, Cattell, and Schaie, which will lead up to extending these models that have attempted to separate effects in factor scores associated with age and cohort. Considered here will be changes in cognitive complexity as viewed from a multivariate perspective (the number of factors and their interrelationships).

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[11] An extended version of this chapter is to appear in Buss (1973d).

Three Factor vs. Two Factor Developmental Model

Schaie's (1965) pioneering article on a general developmental model advocated separating the sources for developmental change into the three components of age, time of measurement, and cohort differences. These three components were viewed as corresponding to maturational, environmental, and either environmental and/or genetic effects respectively. In order to adequately assess the effects of these three variables, Schaie outlined cross-, time-, and cohort-sequential methods. Briefly, each of these methods involves measures at several values on any two of age, time of measurement, and cohorts, thus yielding a possible totality of three bifactorial designs where the third component is always confounded. In the cross-sequential method, cohort and time of measurement are the two variables considered and age is confounded. For the time-sequential method, the variables are age and time of measurement where cohort is confounded, while the cohort-sequential method involves cohort and age thus confounding time of measurement. As Schaie (1965) recognized, the three components of age, time of measurement, and cohort are not independent, since specifying any two completely determines the third. In fact, this property provides the rationale for considering only two of these variables at a time, since a three-way analysis of variance is impossible given that all three variables cannot be independently varied. For meaningful results in any of Schaie's sequential methods, the assumption must be made that the confounded third variable is in fact not related to the dependent measure.

Baltes (1968) has criticized Schaie's formulations on two counts. First, only two components rather than three are required, since having specified say age and cohort, time of measurement is completely determined. The second criticism is that Schaie's attempt to identify age, time of measurement, and cohort differences as directly reflecting maturational, environmental, and either environmental and/or genetic effects respectively, is inadequate, since the former three variables simply classify individuals within the time continuum. Considering Baltes' first criticism, the nonindependence of Schaie's three components was clearly recognized in Schaie's (1965) original paper, and, as noted above, it was this property that necessitated the generation of three separate bifactorial designs rather than a single trifactorial design. More recently, Schaie (1970, 1973) has responded to this criticism by reacknowledging the nonindependence of his three components, as well as correctly asserting that it does make a difference which two of the three components one considers. With respect to the last point, consider, say, a bifactorial design involving age and cohort, where the cells of this matrix are indexed by the completely determined third factor, time of measurement. If one were to then proceed to take age and time of measurement as the independent variables by appropriately rearranging the cells, and indexed the cells with the completely determined cohort factor, this second matrix would involve the same cells as the first matrix, but would be quite different. That is to say, the cells would be regrouped such that time of measurement would yield effects not identical to the previous cohort factor, while the age factor would yield identical effects across this instance of cell regrouping. This example may serve to make the point that, although the three components age, time of measurement, and cohort are not independent, it is still logically possible to set up three separate bifactorial designs in the manner that Schaie has done which will yield different information. One may still question, however, the utility of this different information.

Baltes (1968) has taken the nonindependence property of Schaie's three components as justification for considering only two components at a time, since a simultaneous analysis of the same data using more than one bifactorial leads to severe confounding. For substantive reasons, Baltes recommends one particular bifactorial (age X cohort) for addressing developmental questions. In Baltes' model, he has proposed both longitudinal and cross-sectional sequential data gathering strategies, where the former involves taking two cohorts and sampling across age, and the latter involves taking two adjacent cross-sectional designs where these are represented as diagonals in a cohort X age matrix (see Figure 10 below).

Although the writer agrees with the adoption of Baltes' single bifactorial model, it seems that thus far there has not been sufficient justification for it as an alternative to Schaie's model, since Schaie (1965) has from the beginning recognized the nonindependence of his three components. It would seem that further argument on this matter must involve the interpretation of Schaie's third component, time of measurement, since it is this variable that allows Schaie to consider two 'extra' bifactorial designs. This third variable is one of the two independent dimensions in Schaie's time-sequential and cross-sequential methods. Each of these methods is sutiable only when the assumption can be met that the dependent variable is not related to cohort and age differences respectively. Since most, if not all, developmental questions involve assessing changes on variables that are undoubtedly related to age and cohort, it would seem that Schaie's time and cross-sequential methods would be of little value for addressing developmental questions. It should also be noted that, whereas age- and cohort-related behavioral changes have significant independent meaning, it is difficult to conceive of what meaning time of measurement-related behavioral changes would have apart from translating this into age and cohort effects. It would seem that the intrinsic meaning of the concepts age and cohort would recommend their retention at the expense of discarding the concept time of measurement. That is to say, both age and cohort are independently meaningful when co-considered, as opposed to substituting time of measurement for one of them and then trying not to think in terms of the discarded component (age or cohort).

In order to further clarify the above points, consider the following. In Schaie's (1965, 1970) earlier writings, he considered time of measurement as reflecting an environmental effect, while more recently (Schaie, 1973), he has used the terminology cultural change. Eckensberger (1973) has recently argued for including specific cultural variables in the general developmental models here being considered. Most relevant in the present context is that, in considering a given subpopulation that represents a certain value on a predefined cultural dimension, if there are no cultural changes with respect to that cultural variable, then there can be no nongenetic induced cohort differences within that subpopulation with respect to that cultural variable. This observation serves to illustrate that cohort differences are largely defined in terms of cultural changes (as well as changes in the gene pool). In considering Schaie's time-sequential method, where time of measurement and age are the variables in question, the assumption must be made that there are no cohort effects. This is tantamount to assuming that there are no effects associated with cultural change. Since Schaie (1973) interprets time of measurement as reflecting effects associated with cultural change, this yields a serious contradiction in the time-sequential method.

Further in regard to Schaie's functional interpretation of time of measurement as reflecting effects associated with cultural changes, it should be noted that Wohlwill (1970b), more correctly it seems, interprets this dimension as indicating effects associated with changes of a more situational kind, that is, temporary variations or aberrations in the testing situation. Such effects should be viewed as problems in research design, where such extraneous variables should be controlled for (and therefore not confound the age X cohort bifactorial) by making test conditions uniform. In other words, time of measurement is not a construct of intrinsic interest on a par with age and cohort with respect to developmental questions, and it is best considered as associated with extraneous test variables that should be controlled for by an adequate research strategy. This observation again calls into question the usefulness of the concept time of measurement for generating the cross-sequential and time-sequential methods, as well as Schaie's interpretation of this concept as reflecting effects associated with environmental cultural changes.

Finally on this particular issue, a closer look will be given to how Schaie views the traditional developmental designs. There can be no argument that the simple cross-sectional design confounds age and cohort effects, and the way to unravel these two sources of variance is to employ Schaie's cohort-sequential method or Baltes' age X cohort bifactorial. Schaie views both the simple longitudinal and time-lag (successive sampling of cohorts at the same age) designs as confounding age and environment or cultural change, and cultural change and cohort effects respectively. It should be noted that Baltes considers both of these simple designs as measuring pure age and cohort effects respectively. Assuming uniformity of testing conditions in the time-lag design, where different cohorts at the same age are successively sampled, it would seem that there can be no argument denying that this method taps pure cohort effects. The longitudinal design requires further consideration. Consistent with Schaie's view, it could be argued that the longitudinal design confounds "genetically programmed unraveling of behavior characteristics with cumulative effects of the interaction of organismic states with the environment (Nunnally, 1973, p. 82)". Nunnally (1973) in passing, however, seems to consider this view as not very sensible. It would seem that the most meaningful way to conceptualize the longitudinal method is as follows. During the course of obtaining longitudinal measures, there are undoubtedly genetic and environmental factors operative. In terms of causing a specific behavioral form, both of these influences are 100% important and inseparable. Longitudinal measures for a given cohort may be viewed as pure age-related effects for that specific cohort. In other words, the definition of cohort embodies the idea that there are some specific cultural or environmental changes that occur as age increases. Thus age effects are not to be considered in terms of the absence of the cumulative effects of the environment or cultural changes as Schaie would have, but in terms of a specific cohort (and thus a specific gene pool and a specific sequential pattern of cultural changes). In other words, effects associated with age are relative to a given cohort, rather than being viewed as occurring in some idealized 'normal psychological environment' where the cultural changes, which in part define a given cohort, superimpose their effects on the 'normal aging process'. The former view is especially potent and sensible when considering psychological variables, but also holds for traits of a physical nature. Height of persons, for example, seems to be increasing with each generation, which is due to either genetic and/or environmental (e.g., better nutrition) factors. At which point in historical time should one consider age-related changes in height as occurring in a 'normal' environment -- 500 years ago, now, 500 years hence? This problem may be avoided by considering age-related changes, as obtained by the longitudinal method, as reflecting pure age changes with respect to a given cohort, and thus with respect to a given gene pool and sequential pattern of cultural changes. It should be quickly added, however, that pure age (or cohort) effects, as defined here at the surface level, may still be broken down into deeper source components that account for individual differences. The latter issue has been more closely examined in Buss (1973d), where genetic and environmental variance components were shown to be applicable to both the age and cohort variable.

Changes in Cognitive Complexity

The interpretation remaining to be done with the variables age and cohort does not detract from the value of the recent empirical studies based upon these models. This growing body of research in the abilities domain (e.g. Baltes & Reinert, 1969; Nesselroade, Schaie, & Baltes, 1972; Riegel, Riegel, Riegel, & Meyer, 1967; Schaie & Strother, 1968a, 1968b) and more recently in the personality domain (Baltes & Nesselroade, 1972; Woodruff & Birren, 1972) has found substantial effects associated with cohort differences. This line of research may be formally considered as dealing with changes in factor scores or the _trait profile_ (Anastasi, 1958a). To date, the writer is not aware of any attempts to adapt these models to changes in the factor interrelationships or _trait pattern_ (Anastasi, 1970). In the latter, both changes in the number of ability factors and their interrelationships may be considered as indices for changes in cognitive complexity.

That there is an increase in cognitive complexity associated with ontogenetic change underlies the major theories of cognitive development (e.g. Flavell, 1963; Lewin, 1935; Piaget, 1970; Werner, 1948, 1957). This process of _differentiation_ has served to refer to several aspects of cognitive complexity (Kagan & Kogan, 1970). For present purposes, the idea that it represents an increase in the number of units or dimensions (Bieri, 1961; Kelly, 1955; Lewin, 1935) will be considered. This notion of differentiation may actually be considered as part of the more general view that development is epigenetic in nature rather than preformative (Hunt, 1961, 1969). By epigenesis it is meant that in the course of development, new behavioral forms emerge which have properties not reducible to earlier forms. In the present context, this implies an emergence of cognitive units or dimensions that are not reducible to previous dimensions. Preformation would deny the above by stating that the psychological structure is given at conception!

The concept of differentiation has also received attention from people with a multivariate orientation towards structuring behavioral change in the form of the ability factor differentiation hypothesis (Burt, 1919, 1954; Garrett, 1938, 1946). Evidence for changes in the trait pattern (factor interrelationships), or ability factor differentiation associated with age, has involved such things as an increase in the number of factors, lower interfactor correlations, and a decrease in the percentage of common variance accounted for by the first factor extracted. Recent reviews of the empirical evidence for the ability factor differentiation hypothesis (Anastasi, 1958a, 1970; Baltes & Nesselroade, 1973; Horn, 1968; Reinert, 1970), as well as a theoretical discussion (Royce, 1973a), have generally advanced a cautious affirmation as to its validity.

The inability to muster strong empirical evidence thus far in favor of the differentiation of ability factors during development, appears to be more a function of the subtle methodological pitfalls associated with detecting this process rather than the possibility that it is not a significant phenomenon. This possibility is reinforced both by the observation that the concept of differentiation is central to major developmental theories as noted above, and that recent reviews of the ability factor differentiation hypothesis have been able to construct an almost endless list of deficiencies in design associated with spurious age-related changes. Two important sources producing contaminated age-related changes include the preponderance of cross-sectional designs, which confound cohort effects, and discrepancies in factor analytic techniques.

In considering the two methodological problems mentioned above, a systematic attack on the differentiation hypothesis that makes use of a uniform factor analytic technique and is also able to separate genuine age-related changes from generational or cohort-related changes, will more adequately resolve the extent to which ontogenetic changes in cognitive complexity occur over the life-span. Figure 10 presents a modification of Baltes' (1968) original model, where the cells contain separate R-technique factor analyses (correlating variables across persons at one occasion - Cattell, 1952) rather than persons' scores on factors the nature of which are assumed to be invariant across time. The subscripts refer to cohort and age respectively. Thus, rather than changes in factor scores, changes in factor complexity becomes the unit for analysis as revealed by the number of factors extracted or by the R_{pq} matrix of factor intercorrelations.[12] By applying a uniform factor analytic technique (see Harman, 1967, or Mulaik, 1972) where the criteria for commonality estimates, factor rotation, and the number of factors to be extracted remains constant, one may get a measure of cognitive complexity for each of the cells. Cognitive complexity

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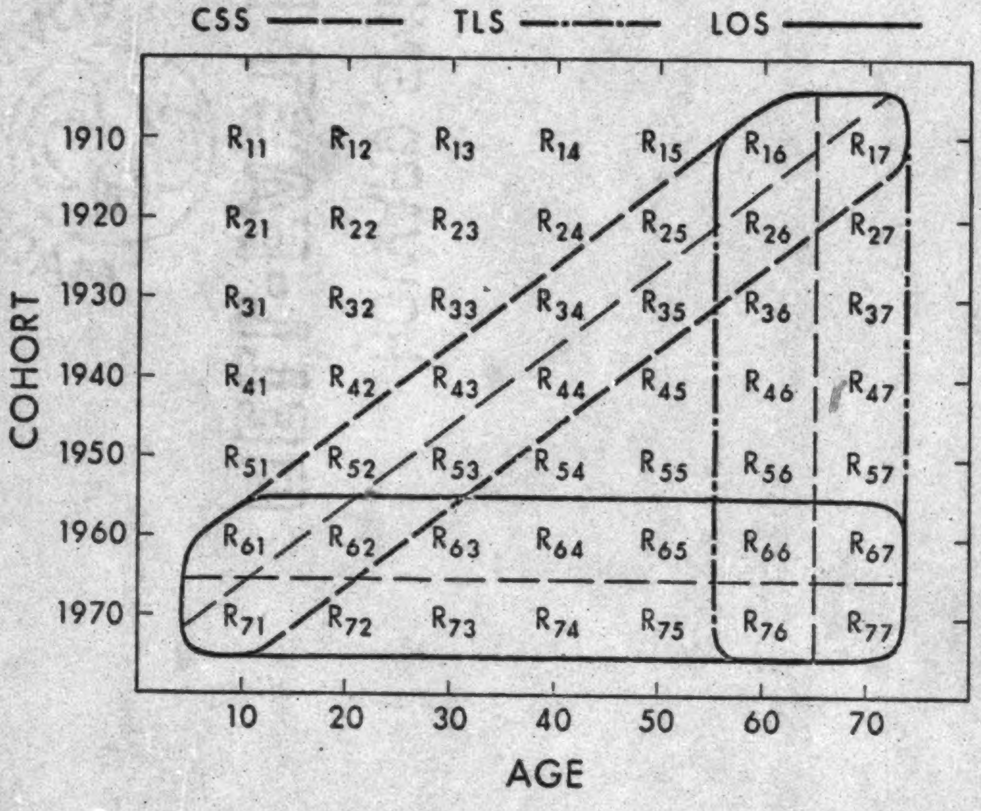
[12] Strictly speaking, in order to adequately assess changes in factor interrelationships, one should factor covariances rather than correlation coefficients, since the former are not subject to spurious changes due to alterations in sampling distributions from occasion to occasion. That is to say, the correlation coefficient involves a restandardization between occasions (it is actually a standardized covariance) that introduces sources of change due to the rescaling of variables. This leads to the necessity of considering factor covariance matrices (C_{pq}) rather than factor correlation matrices (R_{pq}) in assessing cognitive complexity. For a discussion of identifying changes in factor covariances, see Mulaik (1972, p. 358).

Figure 10

A modification of Baltes' bifactorial developmental model, where the dependent measure is separate R-technique factor analyses, and the time-lag sequential (TLS) is added to the cross-sectional sequential (CSS) and longitudinal sequential (LOS) methods.


CSS
TLS
LOS
COHORT
1910
1920
1930
1940
1950
1960
1970
AGE
10
20
30
40
50
60
70
R_{11} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17}
R_{21} R_{22} R_{23} R_{24} R_{25} R_{26} R_{27}
R_{31} R_{32} R_{33} R_{34} R_{35} R_{36} R_{37}
R_{41} R_{42} R_{43} R_{44} R_{45} R_{46} R_{47}
R_{51} R_{52} R_{53} R_{54} R_{55} R_{56} R_{57}
R_{61} R_{62} R_{63} R_{64} R_{65} R_{66} R_{67}
R_{71} R_{72} R_{73} R_{74} R_{75} R_{76} R_{77}

in a given cell may be defined by the number of factors required to account for the common variance, or by the average factor intercorrelation should the number of dimensions remain constant over a given time-span. To the extent that correlations between factors are not affected by changes in the dimensionality, cognitive complexity may be specified as some joint function of the number of factors and their average intercorrelation.

In order to simultaneously consider effects associated with both age and cohort, one of the three sequential strategies should be employed. Each cell would require two independent R analyses in order to get an estimate of the error variance. It would be necessary to hold constant across cells the number of variables to be factored, since this would control for spurious differences in the number of factors extracted. Additional controls would include representative samples that are homogeneous within any generation with respect to educational experiences, general intellectual level, etc. Of course, it would not be desirable to rigorously control such sample characteristics between generations, since cohort effects are actually defined in terms of such population differences.

In considering the possibility of ability factor differentiation over the life-span, it becomes necessary to subscribe to the view that such factors are changing in their basic nature (i.e., different factor pattern matrices or changes in factor loadings). That factors, or variables in general, undergo such structural change over time is generally accepted (Baltes & Nesselroade, 1970, 1973; Coan, 1966, 1972; 1968; Nesselroade, 1970; Nesselroade & Bartsch, 1973). Such changes in factors within the context of differentiation over the life-span, preclude the desirability of carrying out factor matching procedures, since the basic criteria for assessing change (i.e., changes in the number of factors and/or the factor intercorrelations) is contradictory to establishing invariant factors.[13]

In considering the question of changes in cognitive complexity in a truly life-span spirit, there is growing evidence (reviewed by Baltes & Nesselroade, 1973; Reinert, 1970) that an integration -- differentiation -- integration sequence is most appropriate, and is parallelled by early childhood, childhood and adolescence, and adulthood and old age respectively. In terms of the present model, this view could be tested by carrying out trend analyses on the dependent measures. Many developmental theorists, however, view differentiation, and its antithesis, integration, as occurring concurrently rather than consecutively (Bruner, Olver, & Greenfield, 1966; Flavell, 1963; Piaget, 1970; Werner, 1948, 1957). That cognitive development is characterized by increasing differentiation with hierarchical integration, may be interpreted within the factor analytic model as an increase in the number of primary ability factors with a simultaneous increase in the order of higher order factors. The latter are obtained by factoring lower order factors.

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[13] This is not to deny the importance of establishing the identity of factors across different populations that are not defined by age differences (e.g., cultures), as well as across different variables and relatively short term time-spans between occasions. However, the attempt to match factors across populations (same or different individuals) that are defined by substantial age differences, with the attendant concern of devising scaling procedures for the purposes of quantitatively comparing such individuals (Cattell, 1969), is contradictory to ability factor differentiation over the life-span. Indeed, Cattell (1971) and Coan (1966, 1972) have argued, for the ability and personality domains respectively, that the same number of dimensions are present in young children as compared to adults, which would seem to be a prerequisite for that special case of factor matching where the different populations are defined by age differences.

Finally, it should be noted that separating the effects associated with age and cohort for changes in cognitive complexity is descriptive in intent rather than explanatory. Significant effects associated with these variables should serve as an impetus for specifying the nature of the processes that bring about such changes. Variables associated with cohort differences, such as gene pool, educational level, technological advances, sociological and cultural changes, etc., will need to be explored. Concerning age-related processes, the original ability factor differentiation hypothesis (Burt, 1919, 1954; Garrett, 1938, 1946) was interpreted as a maturational phenomenon, while more recently (Anastasi, 1970; Cattell, 1971; Ferguson, 1954, 1956; Horn, 1968, 1970) cultural-learning mechanisms have been considered. Needless to say, both of these forces are operative and necessary for ability factor differentiation.

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CHAPTER 7

A GENERAL DEVELOPMENTAL MODEL FOR INTER-INDIVIDUAL DIFFERENCES, INTRA-INDIVIDUAL DIFFERENCES, AND INTRA-INDIVIDUAL CHANGES[14]

The purpose of the present chapter is to consider, within a developmental context, the interrelationship between certain concepts that have been in the psychological literature for some time. This analysis will result in a general developmental model that will outline several descriptive data gathering strategies. Thus the concepts inter-individual differences, intra-individual differences, and intra-individual changes are defined in terms of sampling across one of the dimensions of individuals, variables, and occasions respectively. The possibility of co-considering a second dimension generates an additional six data gathering strategies. Finally, a simultaneous consideration of the three dimensions of individuals, variables, and occasions yields an additional six data gathering strategies. Out of the total number of 15 data gathering strategies outlined, no less than 11 are defined in part by the occasion dimension and are thus appropriate for addressing ontogenetic or age-related changes.

Before proceding to the main substance of this chapter, it should be noted that many of the ideas which follow have been previously discussed by several people with a multivariate orientation to the structuring of behavior, and especially those who make use of factor analyses. Points of contact between this previous work and the present effort are appropriately indicated below. The major impetus for considering the possible data gathering strategies within the context of inter-individual differences, intra-individual differences, and intra-individual changes, is that previous pertinent multivariate work has typically been cloaked in rather technical terms, and which has, unfortunately, had little impact upon the average developmental psychologist. This state of affairs is probably a result of a combination of factors, including any of the following in varying degrees: the nonavailability of much of this work to those in the developmental field; the lack of explicit attempts on the part of multivariate people to spell out developmental implications of their work for the average developmental psychologist; the mathematical and technical background required for understanding much of the relevant multivariate work is relatively quite demanding; and finally, the active avoidance response that a multivariate perspective often elicits from those who operate within a different framework. The intent of the present chapter is to elaborate on some of the more basic ideas concerning person X variable X occasion data that have been greatly extended by multivariate people, and frame these ideas within the familiar concepts of inter-individual differences, intra-individual differences, and intra-individual changes. The descriptive data gathering strategies to be outlined may be considered truly general, since the dependent variables may consist of measures on any type of variables, (i.e., not necessarily factors) from various domains, and there is no need to undertake a factor analysis at any point in what follows.

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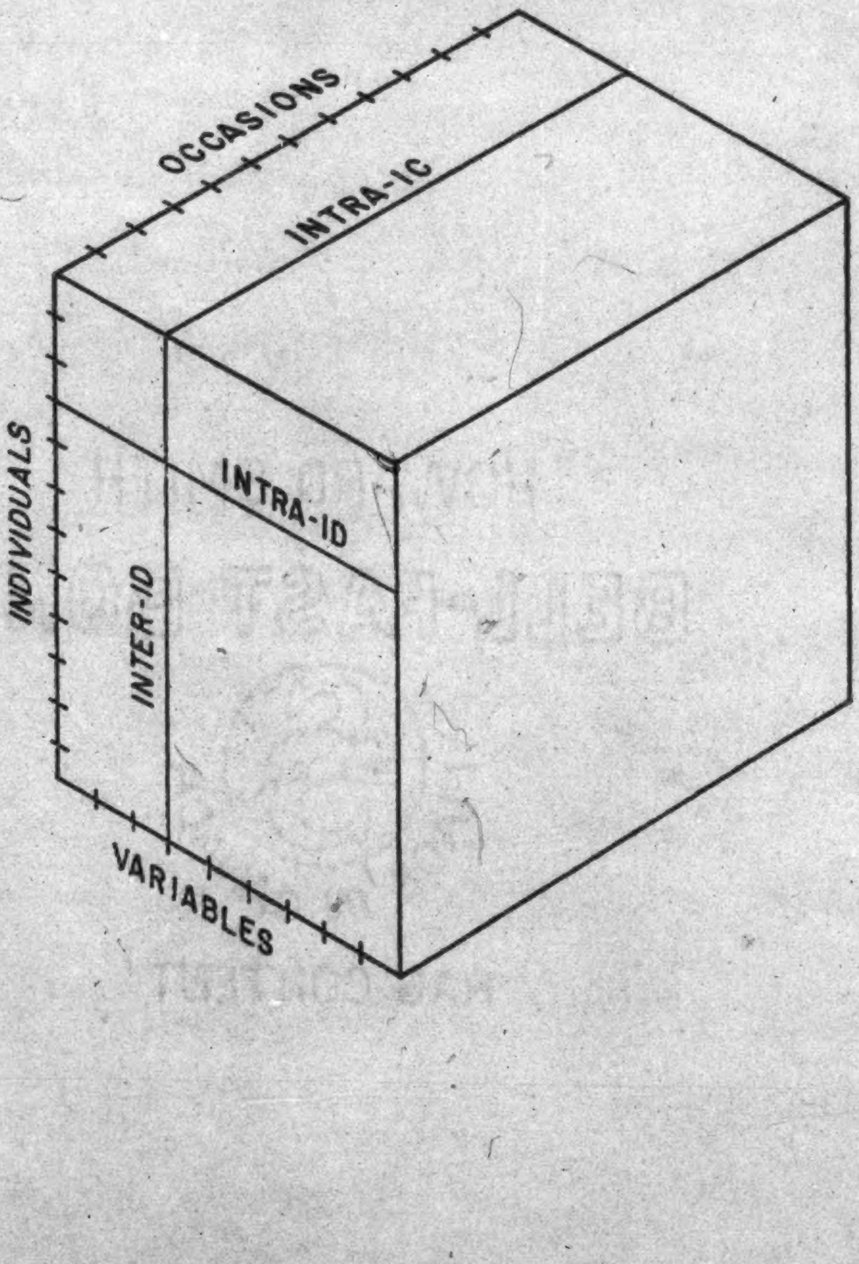
[14] A version of this chapter is to appear in Buss (1973e).

The Model

In Figure 11, the concepts inter-individual differences, intra-individual differences, and intra-individual changes are defined in terms of Cattell's (1946, 1952) three dimensional data frame of

Figure 11

The three cases generated by sampling across each of the three dimensions of individuals, variables and occasions, are respectively inter-individual differences (INTER-ID), intra-individual differences (INTRA-ID), and intra-individual changes (INTRA-IC).


OCCASIONS
INTRA-IC
INDIVIDUALS
INTRA-ID
INTER-ID
VARIABLES

individuals, variables, and occasions. The dependent measure for each cell is an individual's variable score at a particular occasion. Thus inter-individual differences are defined by sampling across individuals for each variable at one occasion. Intra-individual differences are defined by sampling across variables for each individual at one occasion. Finally, intra-individual changes are specified by sampling across occasions for each variable for one individual.

Several comments can be made concerning the model in Figure 11. The main intent of Cattell's (1946, 1952) original model or covariation chart, was to explicate the total possible relations between persons, tests, and occasions, for carrying out correlational researches, although it was stated that other statistical techniques could, in principle, be applied. More specifically, six types of factor analytic techniques were outlined (*R*, *Q*, *P*, *O*, *S*, and *T*) in terms of 'what one correlates over what.' For example, the common *R*-technique involves correlating tests across people at one occasion, while *P*-technique is defined by correlating tests across occasions for one person. In other words, Cattell's covariation chart served the purpose of generating different *kinds* of factors (e.g., test factors, people factors, etc.) by considering different covariance relationships between the components of the three dimensions of tests, persons, and occasions.

More recently, the original covariation chart has been extended by both Coan (1966) and Cattell (1966d), where the former has considered four dimensions (persons, variables, stimuli, and occasions) taken three at a time in various combinations of rows, columns, and entry elements. This procedure generates 24 factor analytic techniques. Coan's (1966) extension pays considerable attention to the relevance of these techniques to developmental psychology, since most of them involve varying the occasion dimension, and thus deal with change data.

Cattell's (1966d) own extension of his original covariation chart can be regarded as the most seminal contribution to setting out the possible data relational systems.[15] In the expanded Basic Data Relation Matrix (or "data box"), a full 10 dimensions[16] (or sets) are used for indexing a particular datum. In considering a given relational system that is defined by a subset of sets, it may be analyzed in different directions, thus implying a somewhat vast number of data gathering strategies. It should be noted that this comprehensive data relational model makes no commitment to a particular statistical method (e.g., factor analysis), since a relational analysis is considered independent of the statistical analysis.

Limiting the present treatment to Cattell's original three sets (individuals, variables, and occasions) which comprise a data frame

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[15] Another important contribution to the theory of data is Coombs (1964), although it is not directly relevant to the present paper. Coombs views data as a relation on either a pair of points or on a pair of pairs of points (or dyads) represented in geometric space. Four sets are explored for indexing a particular datum, i.e. dimensions of an n-dimensional space, trials, and two sets that refer to real world objects (e.g., individuals and stimuli). The four basic kinds of data that Coombs outlines include preferential choice data, single stimulus data, stimulus comparison data, and finally, similarities data.

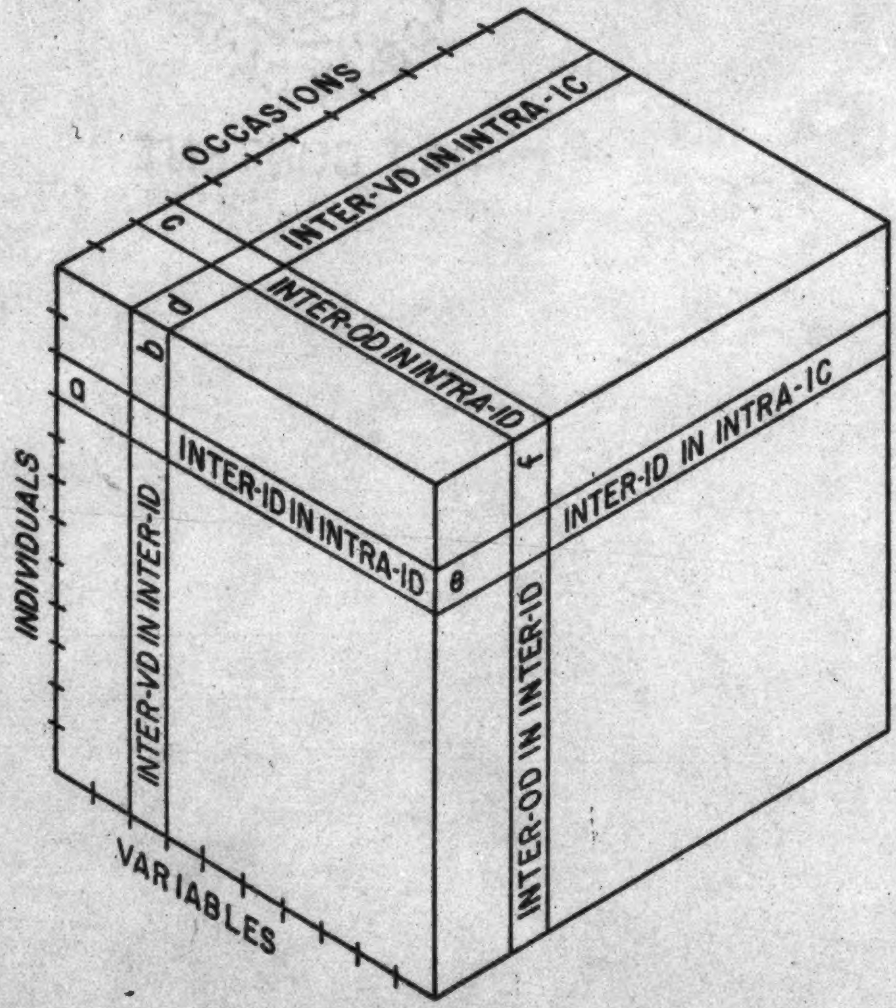
[16] The 10 dimensions include five different types of entities, that is persons, focal stimuli, environmental background stimuli, responses, and observers, and respectively five variants which specify the condition each of these is in, that is states of persons, variants of the focal stimulus, phases of the environmental background stimuli, styles of the responses, and finally, the conditions of the observers. As the reader may have suspected by now, Cattell's (1966d) treatment of data relational systems is quite extensive, comprehensive, analytic, and demanding, and the present brief treatment is unable to do it full justice.

(i.e., more than two sets - Cattell, 1966d), it will now be useful to consider some specific data gathering strategies in terms of familiar concepts, and which will be of particular interest to the developmental psychologist. Figure 12 extends the concepts of inter-individual differences, intra-individual differences, and intra-individual changes by considering the six possible ways of comparative sampling across each of the three dimensions. That is to say, for each of the three dimensions, the simple case is indicated where at least two components or ids (Cattell, 1966d) are sampled across each of the remaining two dimensions or sets. The six cases are: (a) inter-individual differences in intra-individual differences, where individuals are compared in terms of sampling across variables at one occasion; (b) inter-variable differences in inter-individual differences, where variables are compared in terms of sampling across individuals at one occasion; (c) inter-occasion differences (changes) in intra-individual differences, where occasions are compared in terms of sampling across variables for one individual; (d) inter-variable differences (intra-individual differences) in intra-individual changes, where variables are compared in terms of sampling across occasions for one individual; (e) inter-individual differences in intra-individual changes, where individuals are compared in terms of sampling across occasions for one variable; and finally, (f) inter-occasion differences (changes) in inter-individual differences, where occasions are compared in terms of sampling across individuals for one variable.

Each of the above six data gathering strategies is defined by what is compared, which gives the first aspect or the inter-individual differences, inter-variable differences, or inter-occasion differences

Figure 12

The six cases generated by comparative cross-sampling for the simple case where only two components from one dimension are compared in terms of sampling across a second dimension. Abbreviations for the following terms are indicated in brackets: individual differences (ID); individual changes (IC); variable differences (VD); and occasion differences (OD).


OCCASIONS
INDIVIDUALS
VARIABLES
a
b
c
d
e
f
INTER-VD IN INTRA-IC
INTER-OD IN INTRA-ID
INTER-ID IN INTRA-ID
INTER-VD IN INTER-ID
INTER-ID IN INTRA-IC
INTER-OD IN INTER-ID

part, and in terms of what set is sampled across, which gives the second part or inter-individual differences, intra-individual differences, or intra-individual changes aspect. The two inter-occasion comparison cases (c and f) may be considered as changes through time. It may be appropriate to consider the d case as intra-individual differences in intra-individual changes, since different variables are compared in terms of intra-individual changes. This observation reveals that there are two distinct ways of operationalizing the concept of intra-individual differences in Figure 12 -- by sampling across variables for one individual at one occasion (a and c), and by comparing variables in terms of sampling across occasions for one individual (d). Similarly, there are two distinct views of inter-individual differences -- by sampling across individuals for each variable at one occasion (b and f), and by comparing individuals in terms of either sampling across variables at one occasion (a), or by sampling across occasions for one variable (e).

Although each of the six cases in Figure 12 are indicated by only comparative sampling for two ids, the more general case would involve an entire two dimensional matrix or facet (Cattell, 1966d). Thus, cases a and b, for example, would involve the same data from an individual X variable facet at one occasion, but in a one would first sample across variables for each individual and compare the latter, while in b one would first sample across individuals for each variable and compare the latter. A similar situation exists for the remaining four cases in Figure 12.

Since the a and b data gathering strategies are carried out at one occasion, they are not particularly useful for addressing developmental aspects of changes in variable scores, unless one co-considers the third dimension of occasions (see below). In contrast, the remaining four data gathering strategies, which involve in part sequential dependent measures or changes in variable scores through time, would be especially useful developmental paradigms. In considering the two cases that sample across the occasion dimension (_d_ and _e_) and are thus concerned with intra-individual changes, the focus of interest in comparing either variables (_d_) or individuals (_e_) requires that the unit for analysis be the plotting of the entire set of variable scores through time. A useful statistical technique that could be employed here would be testing for trends (Kirk, 1968). This focus on the pattern of changes in variable scores may be contrasted with the other four cases, where the unit of analysis for making comparisons would be variances. In the _a_ case, for example, where inter-individual differences or comparisons are made with respect to intra-individual differences, the appropriate index for the latter is a measure of the within person variance of variable scores at one occasion. In order to compare variables at one occasion in terms of inter-individual differences (_b_), again it is a variance measure that captures the extent of the inter-individual differences. A similar situation holds for cases _c_ and _f_.

Variable scores are typically standardized across individuals for each variable at one occasion. This common practice will need to be avoided in the present scheme, since it would result in identical variances for each variable at each occasion (the variance of a standardized variable is equal to unity). If such a standardizing procedure were adopted, it would be impossible to detect inter-variable differences in inter-individual differences (_b_). By a similar argument, it would be undesirable to standardize each variable across occasions for each person, each occasion across variables for each individual, etc. What is necessary for meaningful comparisons for all six cases is to standardize each variable in terms of both individuals and occasions, i.e., across each rectangular 'slab' or facet for each variable. In this way, spurious identical inter-individual differences variances for each variable at each occasion that are brought about by artificial rescaling procedures, are avoided. One of the advantages of standardizing in the manner being recommended here is that absolute changes in variable scores that result in either higher or lower values will result in correspondingly higher or lower standard scores, since variables are not restandardized within each occasion. The reader would be well advised to work through each of the six data gathering strategies, keeping in mind the standardizing procedure being advocated, in order to realize that such a uniform policy truly permits meaningful results for all six cases.

In Cattell's (1966d) treatment of standardization as it relates to the "data box" (more specifically, a restricted three dimensional version), he outlines three types that are distinguished on the basis of what is standardized. Standardizing over a column of referees (those entities to which the relationship refers) is called normative standardization, over a row of relatives (the things that are to be related) is ipsative standardization, and finally, over a "file" of off-set ids (across the third set or dimension) is known as abative standardization. Most relevant for present purposes is to note that, if one were to adopt any one[17] of these standardizing procedures, where each id from the two sets is standardized across ids from the third set, it would be impossible to obtain meaningful results within the present framework. Thus, rather than standardizing *singly* each column (or row or file), each *entire* individual *X* occasion facet should be standardized for present purposes.

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[17] As Cattell (1966d) has noted, it is normally impossible to double

It is possible to extend each of the six data gathering strategies outlined above in that situation where one also samples *through* the third dimension. In other words, there is a three-step process here, where one first samples *across* the first dimension, *compares* such sampling in terms of the ids or components on the second dimension (the six cases just outlined above), and then proceeds to sample the comparisons of cross-sampling *through* the third dimension. In the complete three-step procedure, two dimensional 'slabs' or facets are compared. This process generates six cases, where those data gathering strategies in Figure 12 are now moderated by a term referring to the third dimension that is sampled through. Figure 13 illustrates the complete three-step procedure for the simple case where two ids (which have been sampled across on one dimension) are successively compared across the third dimension. The more general case would involve successive sampling of entire facets through the third dimension. As before, the latter procedure could involve the same data for various cases, but it is the operational sequence of the three sampling steps that determines the six separate relational systems. The six three-step cases in Figure 13 are: (*a*) *inter-occasion differences (changes in inter-individual*

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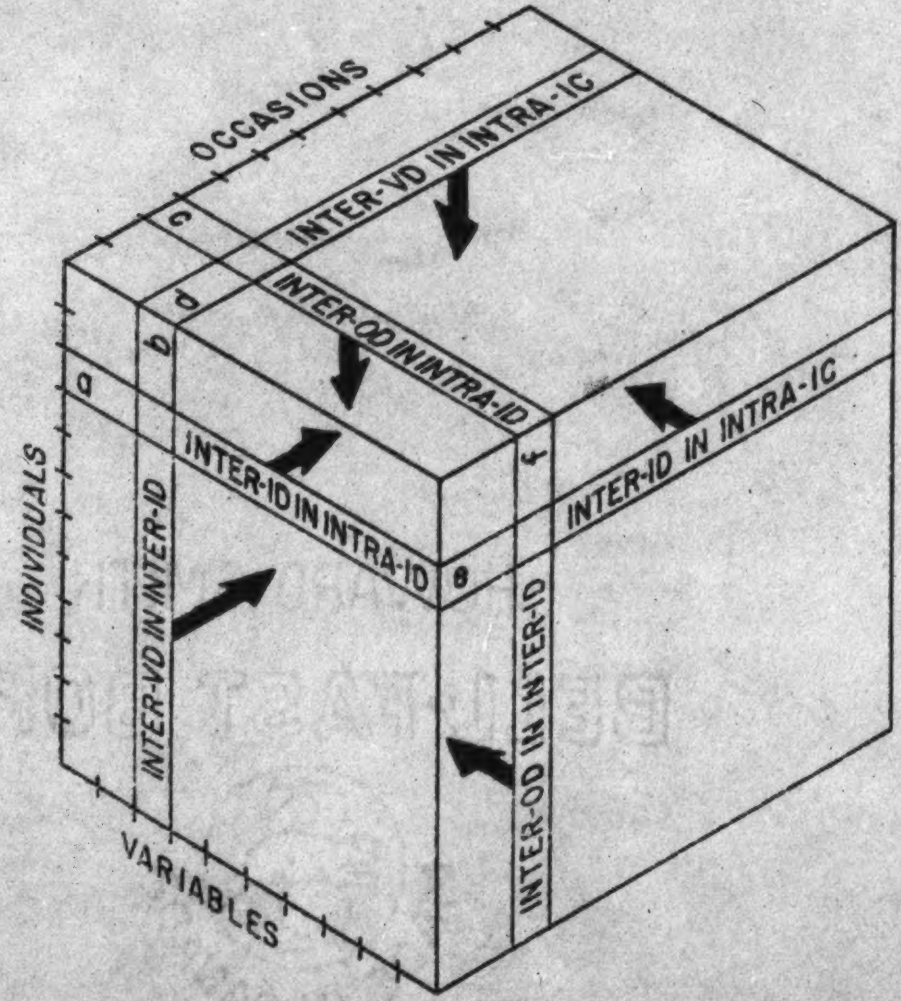
standardize across, for example, both individual rows and individual columns, since the first standardization will be thrown off by the second, and vice versa. Cattell discusses, however, certain procedures that may eventually lead to solving this problem.

Figure 13

The six cases generated by sampling through the third dimension, the labelled comparative cross-samplings. The simple case is indicated, where only two cross-samplings are successively compared through the third dimension. Abbreviations for the following terms are indicated in brackets: individual differences (ID); individual changes (IC); variable differences (VD); and occasion differences (OD).


OCCASIONS
INDIVIDUALS
VARIABLES
c
d
b
a
e
f
INTER-VD IN INTRA-IC
INTER-OD IN INTRA-ID
INTER-ID IN INTRA-ID
INTER-VD IN INTER-ID
INTER-ID IN INTRA-IC
INTER-OD IN INTER-ID

differences in intra-individual differences, where the variances reflecting the extent of intra-individual differences for each individual at an occasion are compared for individuals through time or the occasion dimension; (*b*) *inter-occasion differences (changes) in inter-variable differences in inter-individual differences*, where the variances reflecting the extent of inter-individual differences for each variable at an occasion are compared for variables through time or the occasion dimension; (*c*) *inter-individual differences in inter-occasion differences (changes) in intra-individual differences*, where the variances reflecting the extent of intra-individual differences for each occasion at an individual are compared for occasions through the individual dimension; (*d*) *inter-individual differences in inter-variable differences (intra-individual differences) in intra-individual changes*, where the plots of variable scores across occasions at an individual are compared for variables through the individual dimension; (*e*) *inter-variable differences in inter-individual differences in intra-individual changes*, where the plots of variable scores across occasions at a variable are compared for individuals through the variable dimension; and finally, (*f*) *inter-variable differences in inter-occasion differences (changes) in inter-individual differences*, where the variances reflecting the extent of inter-individual differences for each occasion at a variable are compared for occasions through the variable dimension.

Although these extended six data gathering strategies may appear quite complex *prima facia*, acquiring a firm conceptual understanding of them may be facilitated by working backward through the three steps and, consequently, their verbal designations. For example, in *a* or inter-occasion differences in inter-individual differences in intra-individual differences, the focus is initially upon the extent of intra-individual differences in variable scores at one occasion and one individual as reflected by a variance measure. If one were then to proceed to compare such variances for two individuals at one occasion, we arrive at the two-step concept of inter-individual differences in intra-individual differences. Considering now the third dimension of occasions, where individuals are now compared through time (occasions) in terms of the extent of intra-individual differences in variable socres, we arrive at the three-step concept of inter-occasion differences in inter-individual differences in intra-individual differences. One may work backwards in a similar fashion for each of the six extended cases in order to fully grasp their conceptual significance. The 15 data gathering strategies for inter-individual differences, intra-individual differences, and intra-individual changes, are summarized in Table 3 in the order presented above, where the first three cases consider multi-ids on only one dimension, the next six cases on two dimensions, and the last six cases on all three dimensions.

Discussion

Brief recognition should be given at this point of the many attempts on the part of multivariate people to deal specifically with individual X variable X occasion data, where the aim has been to derive factors within the context of change. Some of these models involve observing both changes in factor loadings and factor scores across time (Baltes and Nesselroade, 1970, 1973; Bentler, 1973; Cattell, 1970b; Corballis & Traub, 1970; Nesselroade, 1970; Nesselroade & Bartsch, 1973). Other models specify data on separate occasions by either a constant matrix of factor scores and separate occasion matrices of factor loadings

Table 3

Data Gathering Strategies for Inter-Individual Differences, Intra-Individual Differences, and Intra-Individual Changes

Dimension 1 Sample Across	Dimension 2 Compare On	Dimension 3 Sample Through	Type
individuals	no	no	inter-*ID*
variables	no	no	intra-*ID*
occasions	no	no	intra-*IC*
variables	individuals	no	inter-*ID* in intra-*ID*
individuals	variables	no	inter-*VD* in inter-*ID*
variables	occasions	no	inter-*OD* in intra-*ID*
occasions	variables	no	inter-*VD* in intra-*IC*
occasions	individuals	no	inter-*ID* in intra-*IC*
individuals	occasions	no	inter-*OD* in inter-*ID*
variables	individuals	occasions	inter-*OD* in inter-*ID* in intra-*ID*
individuals	variables	occasions	inter-*OD* in inter-*VD* in inter-*ID*
variables	occasions	individuals	inter-*ID* in inter-*OD* in intra-*ID*
occasions	variables	individuals	inter-*ID* in inter-*VD* in intra-*IC*
occasions	individuals	variables	inter-*VD* in inter-*ID* in intra-*IC*
individuals	occasions	variables	inter-*VD* in inter-*OD* in inter-*ID*

Note: Abbreviations for the following terms are indicated in brackets: individual differences (*ID*); individual changes (*IC*); variable differences (*VD*); and occasion differences (*OD*)

(Fleishman, 1954; Fleishman & Hempel, 1955; Harris, 1963), or a constant matrix of factor loadings and separate occasion matrices of factor scores (Corballis, 1965; Horst, 1965). Tucker's (1963) multimode method is able to handle data that is indexed by three dimensions, and permits detecting person, variable, and occasion factors simultaneously. Two excellant reviews of most of these models have been provided by Bentler (1973) and Nesselroade (1970).

It has recently been advocated that at the very core of any study of psychological development should be an approach that is concerned with inter-individual differences in intra-individual changes (Baltes & Nesselroade, 1973; Wohlwill, 1970a). In the present framework, this view may be considered as only one of four possible two-step approaches that are capable of dealing with developmental phenomena, i.e., that are defined in part in terms of the occasion dimension. The addition of the six three-step cases, which are all in part defined by the occasion dimension, gives the developmental researcher a wide variety of data gathering strategies from which to choose.

It is possible to work through the various data gathering strategies in terms of groups of individuals rather than single individuals. In Cattell's (1966d) terminology, this could be viewed as a modified (three dimensions rather than two) face, where the data entries (means) are values obtained by condensing a whole series of off-ids (individuals). This approach permits considering inter-group differences, intra-group differences, and intra-group changes in all of the two- and three-step paradigms from Table 3. Such groups may be defined by age differences, sex differences, cultural differences, personality differences, ability differences, etc. For that situation where the groups are differentiated by age differences at a particular occasion, and one sampled through the occasion dimension in the various ways set out, this may be viewed as a cross-sectional sequential strategy (Baltes, 1968), where a given cross-sectional sample is sequentially sampled through time. Where the groups are defined in terms of differences from one domain (e.g., personality, ability, or motivation) and the dependent measure is variable scores from another domain, the question at issue would be the relationship of one domain to changes and/or differences in another domain. It would also be possible to define the groups in terms of a variable from a given domain, and adopt variables from that same domain as the dependent measures.

If one were to consider a special type of variables in the present scheme, namely factors, this would serve to accent the general requirement that, across occasions (or groups) the variables must be conceptually identical, that is, measure the same construct. With respect to factors, there are a myriad of problems associated with establishing their structural invariance across time and/or across groups (Cattell, 1969). Since it has already been noted that the nature or meaning of factors may change across time or groups as revealed by "real" changes in the factor loadings, this will preclude considering, in the present model, factors as variables, except in that instance where structural invariance has been demonstrated. The latter condition will most likely obtain when the occasions are over relatively short term time-spans and/or the groups are drawn from a relatively homogeneous population.

In conclusion it should be noted that the purpose in this chapter has been to explicate the concepts of inter-individual differences, intra-individual differences, and intra-individual changes in terms of the three dimensions of individuals, variables, and occasions. In this way, an array of descriptive data gathering strategies has been generated which may be of particular interest to those in the field of developmental psychology. In the following chapter, a recursive-nonrecursive factor model is elaborated within the context of mapping out developmental causal networks.

CHAPTER 8

A RECURSIVE-NONRECURSIVE FACTOR MODEL AND DEVELOPMENTAL CAUSAL NETWORKS

In the present chapter, a general model for recursive-nonrecursive causal networks of factors will be outlined. The technical machinery required for operationalizing the model, in terms of analyzing empirical data, will be briefly indicated. Finally, some of the more recent existent multivariate developmental theory will be briefly noted within the present scheme.

The Model

Figure 14 (a) sets out the causal relationships between factors and a performance variable as implied by the causal structural equation of the type [1], where the symbol for factor scores is now an upper case X which will prove advantageous in this chapter. Here it can be seen that factors X_1 and X_2 determine performance on variable x_1. For ease of presentation, the factor loadings or b's have not been inserted to specify the value for each of the determining 'arrows', as well as the exogenous influences that are 'extra' the endogenous factors that comprise this particular model. Figure 14 (a) represents a recursive model, where the direction of causal determination is one way, i.e., there is no provision in the system for mutual determination. In Figure 14 (b), mutual determination at the level of the variables x_1 and x_2 is indicated by the two-way directional arrow. This represents a nonrecursive causal property between the two variables.[18] In this situation, performance on variable $\underline{x}_1$ is an additive function of the weighted scores on factors $\underline{X}_1$ and $\underline{X}_2$ as well as variable $\underline{x}_2$. A similar statement can be made for variable $\underline{x}_2$. It is possible, of course, to specify say variable $\underline{x}_1$ in terms of factors only, i.e., the $\underline{x}_2$ variable component that would appear on the right side of [1] could be translated into factors $\underline{X}_3$ and $\underline{X}_4$. The interpretation of this situation would be that the causal structure of variable $\underline{x}_1$ is now represented completely in terms of 'ultimate' determiners.

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[18] The definition of recursive and nonrecursive is taken from Van de Geer (1971), who points out the counter intuitive use of these terms. In other contexts, the term "recursive" implies that a certain state x at a

Although Figure 14 is presented in terms of the factor analytic model, i.e., factors determine variables, it is possible to view this model as a special case of path analysis. Path analysis attempts to get at the causal structure of a set of variables where these variables are ordered in time. In dealing with factors as well as traditional variables, and if one were to extend the causal network over time (see below), it can be appreciated that the argument for factors as determiners may find considerable support from the path analysis model. The formal similarities between factor analysis and path analysis have been noted by Van de Geer (1971).

The model to be presented is an extension of Figure 14 (b), i.e., additional factors are considered at later points in time. This provides for a type of path analysis of factors and variables in order to map out developmental causal networks. Figure 14 (b) may actually be viewed as a recursive-nonrecursive model, since there are both

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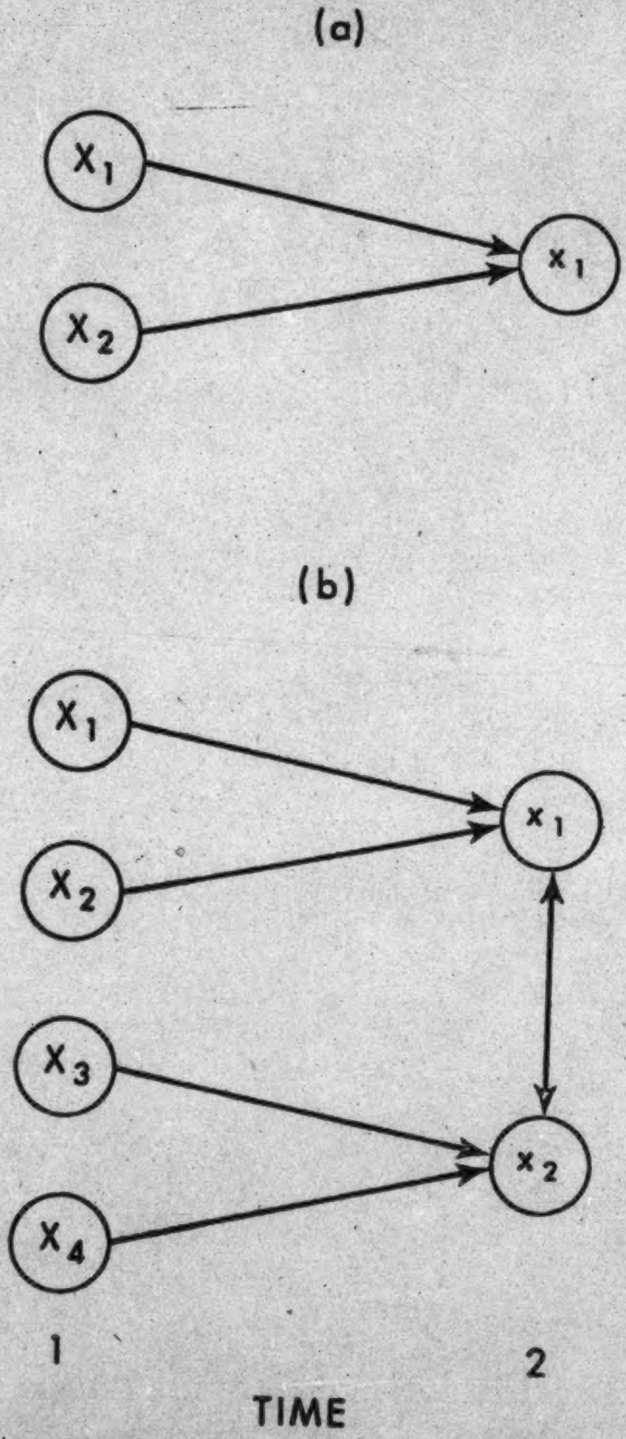
given point in time is a function of state $\underline{x}$ at an earlier point in time (as well as other variables). Van de Geer's use of recursion would not be inconsistent with the latter view (since unidirectional causation could involve a variable that is a function of itself at a later time), but he fails to mention the richer meaning of recursion.

Figure 14

An example of a recursive (a) and a recursive-nonrecursive (b) causal situation.


(a)
X_1
X_2
x_1
(b)
X_1
X_2
x_1
X_3
X_4
x_2
1
2
TIME

recursive and nonrecursive properties built into the system. Three distinct aspects of the recursive-nonrecursive model are considered below. These include: within vs. between domain nonrecursive aspects; within vs. between domain recursive aspects; and finally, a further distinction to be made within the within domain recursive case consisting of different vs. same factors. Figure 15 summarizes the six cases thus generated, where the cells are indexed in the order that they will be considered.

Figure 16 (a) and (b) represent a sequential causal network for cases 1 and 2 respectively. In Figure 16 (a), all the factors at time t_1 are from the same domain (e.g., ability, personality, or motivation factors), the variables at time t_2 are within this same domain, while the factors at time t_3 are from a different domain. Given this situation, along with the causal relationships indicated, case 1 may be called the between domain recursive-within domain nonrecursive model, since there are recursive properties from one domain to another ($X_1 \rightarrow x_1 \rightarrow Y$, etc.), as well as a nonrecursive property within a domain ($x_1 \leftrightarrow x_2$). The interpretation of Figure 16 (a) beginning at the top left hand side, would be that, factors X_1 and X_2 at time t_1 determine variable x_1 from the same domain at time t_2. Variable x_1 is also determined by (and determines) variable x_2. Factors Y_1 and Y_2 at time t_3, which are from a different domain, are determined by variable x_1. Of course there would be exogenous determining influences that would contribute to each of the factors and variables in this model (e.g., factors in the Y domain would also be determined by variables and/or factors from that domain), although they are not represented in this particular causal network. If one were to specify the determinants of

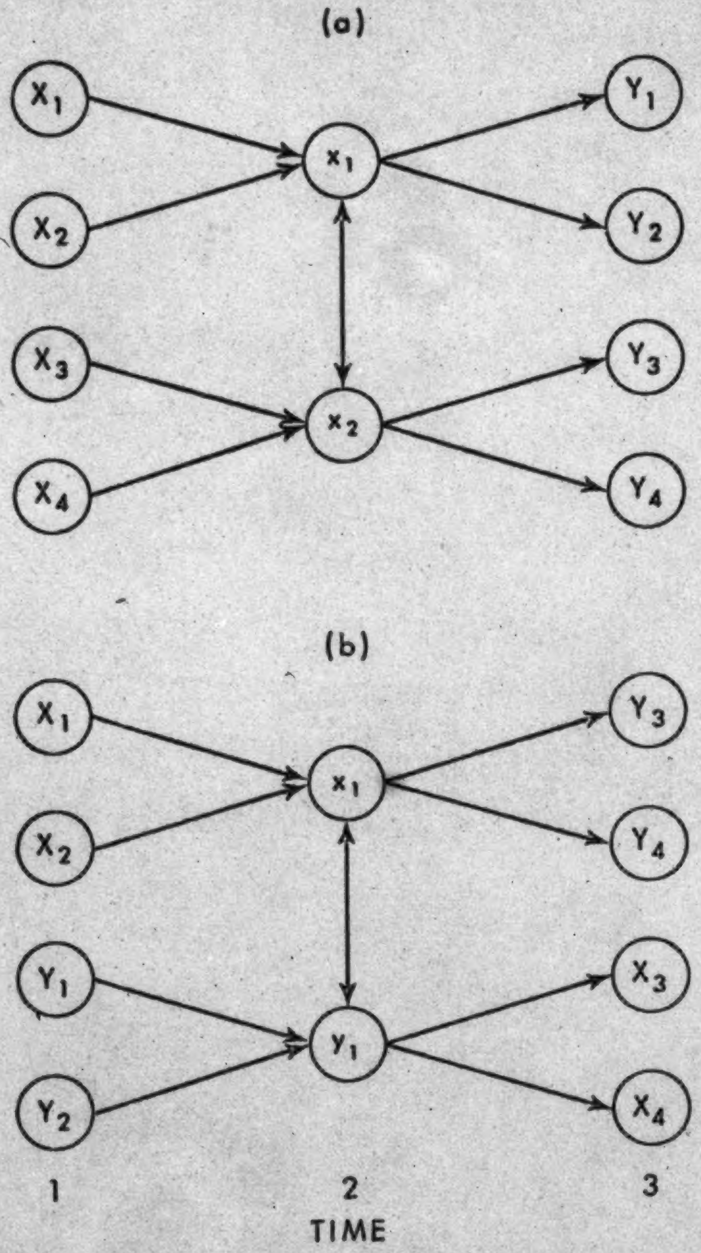
Figure 15

The possible types of recursive-nonrecursive models.

			NONRECURSIVE	
			between domain	within domain
RECURSIVE	between domain		2	1
	within domain	different factors	4	3
		same factors	6	5

Figure 16

A between domain recursive-within domain nonrecursive model (a) and a between domain recursive-between domain nonrecursive model (b).


(a)
X1
X2
X3
X4
x1
x2
Y1
Y2
Y3
Y4
(b)
X1
X2
Y1
Y2
x1
y1
Y3
Y4
X3
X4
1
2
3
TIME

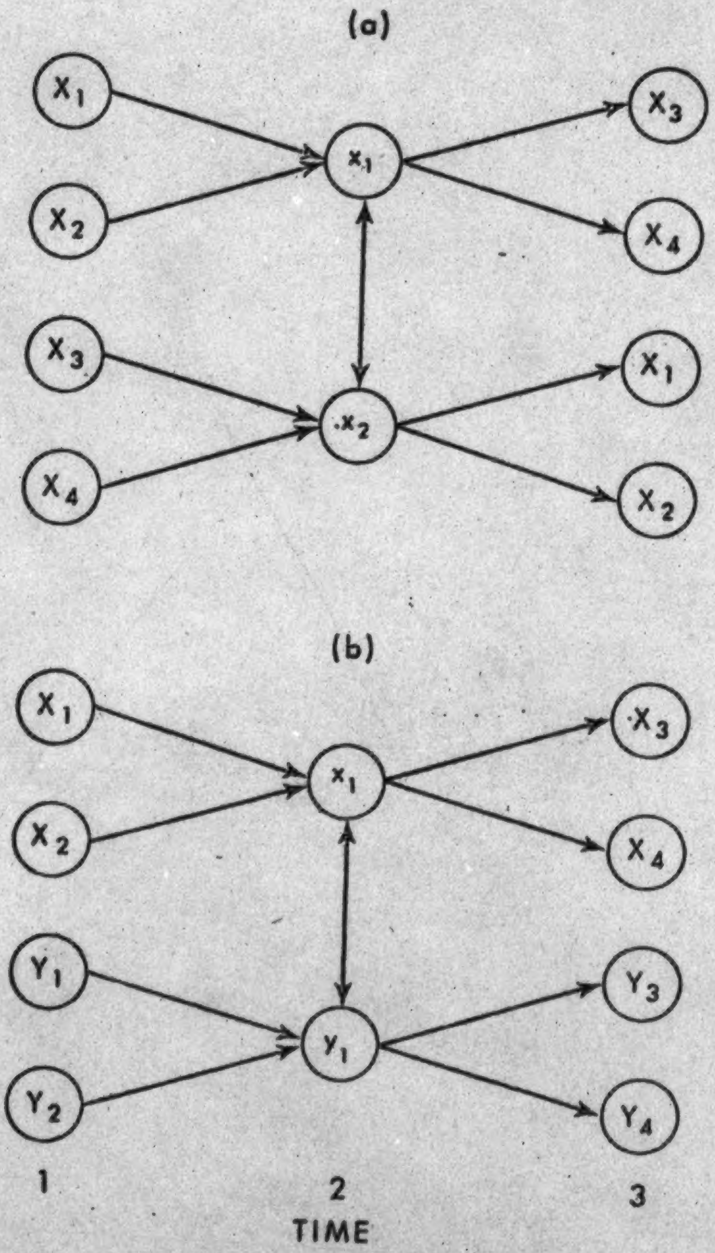
any of the factors at time t_3 in terms of ultimate influences, this would involve constructing a specification equation like [1] for each of the Y factors in terms of the four X factors.

Figure 16 (b) represents a between domain recursive-between domain nonrecursive model. In this situation, factors or variables from different domains are present at each point in time, providing for a nonrecursive property between domains ($x_1 \leftrightarrow y_1$). Thus the variables at time t_2 are specified by both within and between domain sources of influence, while the same holds true for the factors at time t_3 if one were to consider the ultimate determiners at time t_1.

Figure 17 (a) and (b) illustrate within domain recursive-within domain nonrecursive and within domain recursive-between domain nonrecursive models respectively. In both of these cases, the factors at time t_3 are from the same domain as those at time t_1, although each of the two pairs of factors in a recursive relationship with each of the variables at time t_2 are different from each other. Interpreting Figure 17 (a) and, say, factors X_3 and X_4 at time t_3, they are immediately determined by variable x_1 from the same domain, or in ultimate terms, by factors from the same domain (including those factors themselves, i.e., X_3 and X_4 at time t_1). In considering Figure 17 (b) and factors X_3 and X_4, these factors may be viewed as being ultimately determined by the four factors from two domains at time t_1, given the indicated relationships. This observation would seem to contradict part of its definition, i.e., that it is within domain recursive. However, the between domain recursive aspect in terms of ultimate source determiners is actually mediated by the nonrecursive property ($x_1 \leftrightarrow y_1$), and therefore does not alter the original interpretation, especially

Figure 17

A within domain recursive-within domain nonrecursive model (a) and a within domain recursive-between domain nonrecursive model (b), where the factors on each side of the variables at time $\underline{t_2}$ are different.


(a)
X_1
X_2
X_3
X_4
x_1
x_2
X_3
X_4
X_1
X_2
(b)
X_1
X_2
Y_1
Y_2
x_1
y_1
X_3
X_4
Y_3
Y_4
1
2
3
TIME

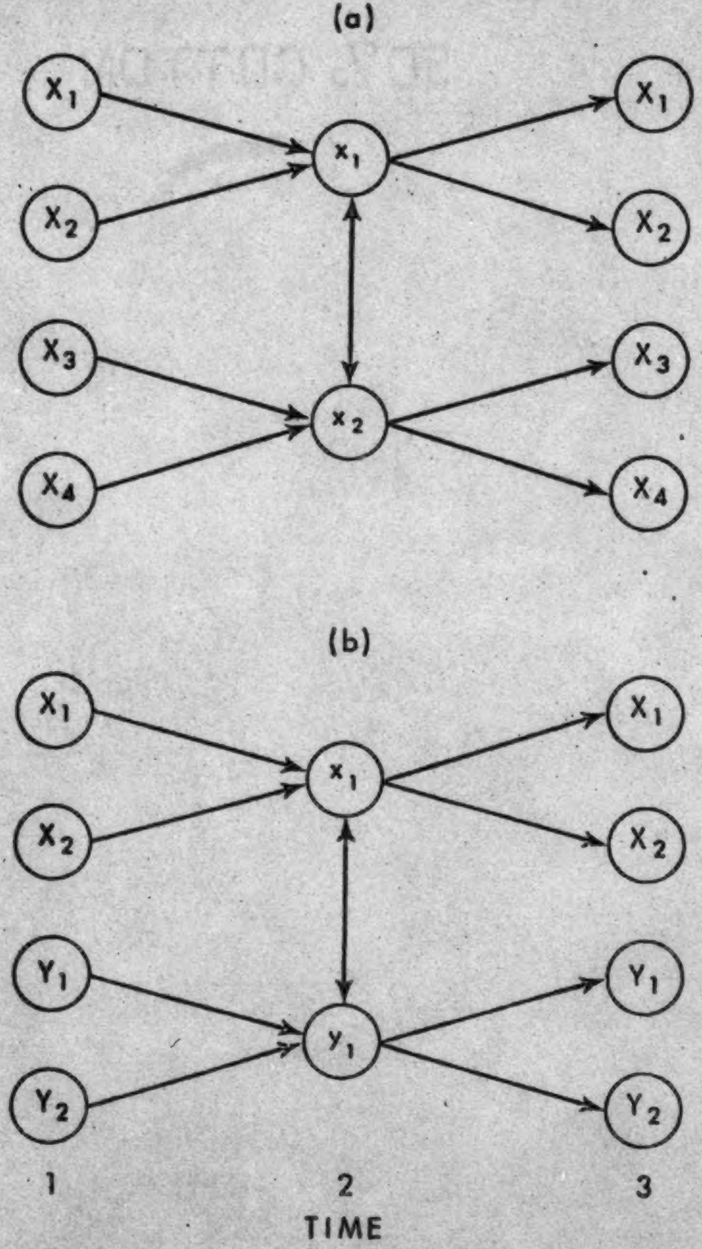
when considering immediate influence sources.

Figure 18 is identical to Figure 17 except for the arrangement of the factors at time t_3. In Figure 18 (*a*), the within domain recursive-within domain nonrecursive model has the *same* factors on each side of variables x_1 and x_2, while a similar situation exists for the within domain recursive-between domain nonrecursive model in Figure 18 (*b*), i.e., one side is the mirror image of the other side. The interpretation here would be that variables at time t_2 are determined by certain factors as well as a within (*a*) or between (*b*) domain variable. These variables in turn determine at a later time t_3 those same factors that originally partially specified those variables. Reduced to the essentials, we have here a situation where a factor determines a variable which in turn determines that same factor at a later point in time (e.g., $X_1 \rightarrow x_1 \rightarrow X_1$). This looks suspiciously like an additional nonrecursive property, given the identities and relationships of the factors in this formally recursive situation (unidirectional arrows). It is the time sequence, however, which distinguishes this recursive situation that masquerades as nonrecursive, from the genuine nonrecursive situations that exist at time t_2 in Figures 16-18. In the latter case, two variables mutually influence each other *at the same point in time*. This simultaneous mutual deterministic relationship does not exist in the recursive aspects of Figure 18 (*a*) and (*b*).

Having formally set out in a simplified manner the various types of recursive-nonrecursive factor models for abstracting developmental causal networks, it should be noted that there is considerable flexibility in terms of interpreting the various components. Although factors and variables from two domains have been considered in distinct

Figure 18

A within domain recursive-within domain nonrecursive model (a) and a within domain recursive-between domain nonrecursive model (b), where the factors on each side of the variables at time t_2 are the same.


(a)
X_1
X_2
X_3
X_4
x_1
x_2
X_1
X_2
X_3
X_4
(b)
X_1
X_2
Y_1
Y_2
x_1
y_1
X_1
X_2
Y_1
Y_2
1
2
3
TIME

time sequences, i.e., variables always occurred at time $\underline{t}_2$, it is possible to have situations where variables occur at times $\underline{t}_1$ and $\underline{t}_3$. The variables may also be replaced by within or between domain factors. One could also consider the variables as higher order factors, or, complex criterion performance variables that cut across domains (e.g., school or job performance). These are only some of the possible interpretations of the components in these models, and they are considered subsequently in terms of current multivariate developmental theory. The observation that the components in the preceeding models may assume diverse identities serves to reinforce the point that they are indeed general models for mapping out developmental causal networks.

Before turning to the psychological interpretations of the recursive-nonrecursive models, brief mention should be made as to the technical procedures required for operationalizing the mapping out of developmental causal networks as described above. Van de Geer (1971) has summarized two techniques for analyzing nonrecursive models with recursive aspects, which may be formally considered as similar enough to the models discussed above such that these techniques would be applicable within the present framework. These techniques were originally discussed by Goldberger (1964) within the context of econometric theory. Because of the nontechnical nature of the present text, as well as the excellent readily available summary by Van de Geer, these computing procedures will not be reviewed here.

Discussion

That factors may be especially useful theoretical constructs when their inter-relationships are considered was first noted in Royce's (1963) pioneering article. What Royce did was to entertain the idea that a given structure of factors may be conceived as specifying a particular nomological net, i.e., an interlocking system of constructs which constitute a theory. In this view, higher order factors are considered further removed from the empirical data and thus penetrate much deeper within the nomological net. Such factors constitute broad, explanatory constructs which account for a relatively larger number of lower order factors.[19]

Cattell (1965) has greatly extended the idea that inter-factor relationships may produce useful theoretical models. Critical here is Cattell's concept of a general reticular (netlike) model, where factors as influences are allowed to operate upon one another in all manner of directions. This means that factors may influence other factors within or between domains as well as within or between factor levels or orders. Nonrecursive properties are dealt with in what are called two-way or feedback models. No attempt is made in these models, however, to explicitly build in the time dimension as has been done in the recursive-nonrecursive models above.

Because of the lack of sufficiently sophisticated data analyzing procedures at the time, Cattell (1966b) recommended that we should "put the general reticular model 'on the shelf' for more advance study", and confine our attention to the one-way stratified model, where factors at higher levels or strata determine lower strata factors. In light of the recent sophisticated procedures for handling nonrecursive models as developed within econometrics, it may be time to re-evaluate Cattell's recommendation. Indeed, more recently, Cattell himself (Cattell, 1971) has ventured further (and once again alone) into this area by speculating on nonrecursive or two-way causal interpretations for both between and within domain situations as well as across different factor stratas. Briefly, this view revolves around what Cattell calls the triadic theory of abilities, which includes three distinct types of abilities, i.e., capacities such as fluid intelligence and general speed, provincials such as visualization and auditory organization, and agencies which include the traditional primary ability factors. It is hypothesized here that there is a two-way causal relationship between the capacities and provincials, as well as between the agencies and the crystallized intelligence factor. It would seem that if one were to explore these relationships systematically, it would be advantageous to adopt the within domain recursive-within domain nonrecursive model with the attendant possibility for operationalizing the sequential data gathering steps. Cattell also considers several instances where personality factors may influence abilities and where ability factors may influence personality during development. Any of the models that consider between domain influences would be appropriate for examining these issues.

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[19] One criticism of higher order factors is that since the unique variance that is discarded is actually common variance at the first order, higher order factors "explain" even less than the primaries. One could respond to this criticism by stating that as one move up the factor strata a smaller and smaller "core meaning" is arrived at which should not be expected to account for the common variance at the test level as well as the first order factors. Another point that should be mentioned here is that the observation of time sequences would greatly enhance the idea that higher order factors determine lower order factors. An example here would be in the ability domain where "g" prevails at lower age levels and the primaries at later age levels (see Chapter 6 and the differentiation of abilities).

The specification of complex criterion variables such as school performance and creativity has been extensively considered by Cattell (1971) and Cattell and Butcher (1968). Critical here is the idea that each of the domains of personality, abilities, and motivation, contribute in approximately equal amounts to performance. In considering developmental aspects of creativity and school performance, Cattell distinguishes between present action and developmental equations. The latter are viewed as specifying the cumulative effects of past experience to present performance level. It would seem that a sequential analysis through time in a manner outlined above may provide the means for operationalizing the notion of developmental structural equations.

In conclusion, it should be noted that the general recursive-nonrecursive factor model outlined here may prove quite useful for mapping out developmental causal networks. This operational model has the advantage of simultaneously dealing with recursive and nonrecursive causal properties, as well as having time sequences explicitly built into the system.

EPILOGUE

Given the nature and format of the present dissertation, the various issues involved in changes in ability factors have been examined and discussed in each of the various preceeding chapters. It is not the intent at this time to review these major issues and conclusions, since each of the above chapters can speak for themselves. The writer would like, however, to end this manuscript on a more personal note, affirming his belief in the importance of the preceding issues for establishing the foundations for a multivariate developmental psychology. That is to say, three separate classes of foundational issues have been examined here with respect to multivariate developmental psychology: the integration of substantive learning and developmental theory (Chapters 3, 4, and 5), meta-theoretical issues or the conceptual framework (Chapters 2 and 8), and finally, methodological considerations or descriptive data gathering strategies (Chapters 6 and 7). The way is now a little clearer in order to begin a truly multivariate developmental psychology.

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APPENDIX A

In this section, a brief consideration will be given to some of the scaling problems involved in detecting "real" changes in factors, where the central problem revolves around equivalence of scale across time. No definitive solutions are arrived at in this regard, but rather some of the problems and relevant work is noted thereby indicating the direction that such solutions may take.

Consider Equations [3], [4], and [5] in Chapter 2 where variable x on the left hand side is in standard score form. More specifically, a change in variable x as indicated by $z^*_{xi} - z_{xi}$ implies that either the raw score, mean, or standard deviation at occasion o_2 has changed, since $z_{xi} = x_i - \bar{x}_i/s_x$. In order to control for spurious changes in z_{xi}, the variable x should be standardized from the pooled scores across occasions, i.e. one obtains a single combined set of standardized variable scores thereby achieving a common mean and variance.

Controlling for differences in metric or variances across occasions (or groups) is a distinct advantage of cannonical, maximum likelihood, image, and alpha factor solutions, since these methods involve rescaling the reduced correlation matrix R by either the reciprocal of the uniqueness (the first three methods) or the reciprocal of the commonality (the last method). All of these procedures will yield identical within method solutions under conditions where there are metric differences for the same variables across occasions.

Recently Bentler (1973) has reviewed the major issues involved in dealing with change data that is factor analyzed. The data of interest here consists of at least two persons X variable matrices obtained on different occasions. Following Bentler's (1973) discussion and terminology, the separate standard score method, which involves obtaining separate factors in the usual manner for each of the data matrices, is found to be quite inadequate for the purposes of reflecting change. Besides the usual problem of establishing the identity of factors from occasion to occasion, this method is unable to reflect real changes in factor scores, since factor scores are standardized within occasions. This approach actually assumes that the means for factor scores are equal across occasions (means of 0), thus precluding the possibility for change.

In the separate raw score method, one factors the raw scores themselves rather than standard scores. Thus information about differences in means and variances is retained. Since such information will strongly affect the loadings across occasions and thus make factor matching quite difficult, it is desirable to normalize the loadings for each factor and thereby absorb the differences in scale and mean in the factor scores rather than in the factor loadings. The latter procedure, however, will not permit the detection of real changes in factor loadings. Another problem with factoring raw scores is that communality and uniqueness become confounded.

The within-group covariance method arrives at a single factor loading matrix for the data from two occasions while allowing the factor scores to change, and would thus be appropriate for reflecting quantitative change (Equation [3]). However since deviation scores are factored, this method must make the questionable assumption of equal means across occasions. In the simultaneous analysis method, in which raw scores are factored rather than deviation scores, information regarding means is retained. However to the extent that it is conceptually desirable to allow for changes in factor loadings, these two approaches must be abandoned since they preclude such a possibility.

In the identical factor score method, as one might guess, factor scores are assumed to be invariant across occasions while factor loadings are permitted to change, and thus would be appropriate for reflecting structural change (Equation [4]). To the extent that one wishes to allow for both changes in factor loadings and factor scores, this method must be abandoned.

The most recent attempt to provide a model in which changes in both factor scores and factor loadings is possible, and would thus permit identifying quantistructural change (Equation 5), is longitudinal factor analysis (Corballis and Traub, 1970). As the authors have noted, the mean factor scores on both occasions, however, are 0, thereby not permitting any mean growth from occasion to occasion. A more generalized version of this model is presented by Bentler which does allow for mean change across occasions.

Cattell (1970b, 1972) has also been concerned with the problem of eliminating spurious changes in factors when one compares groups differing in either age or culture. In the equipotent method, the data of interest across occasions (or groups) are combined for purposes of obtaining a single standard score distribution. This approach thus avoids spurious changes in factor scores that would result if one were to restandardize each occasion (or group) separately. An additional feature of this method is that the weights used for estimating a factor score (from a factor estimation matrix), are rescaled by the reciprocal of the squared multiple correlation of the variables and predicted factor scores. In other words, the loadings used for estimating the factor scores are brought to equipotency or equal footing by a normalized factor estimation matrix. This procedure permits differences in means, sigmas, and individual factor scores to emerge. In the *isopodic* method, Cattell introduces the concept of *real base* factor analysis, in which arbitrary metric differences in variables is avoided by sampling from a *universe of behavior variables*. The raw scores obtained from such sampling are then employed in covariance factoring, where it is assumed that a normalized covariance factor pattern matrix will apply across all groups. In other words, this procedure will yield 'real' changes in factor scores where such scores may differ in mean and sigma across groups, although the factor loadings will be constant across groups. The latter requirement is based upon the tenuous assumption that a given factor will *always* affect the same variables in an identical manner across different groups. Cattell himself, however, implies that this assumption will not hold when the groups to be compared are substantially different. This criticism also applies to Rozeboom's (1971) recent change model, since it likewise assumes invariant factor loadings across time (or across groups defined in terms of time).

Finally the problem of establishing factor invariance for that case where the same variables are factor analyzed in different populations will be briefly mentioned. In the present context, where the focus is upon the same individuals across occasions, each of the occasions may be considered as defining a different subpopulation (in terms of time) although the same individuals are represented at each occasion. Mulaik (1972, pp. 340-360) has presented an excellent summary of the problem of factor invariance under selection of different experimental subpopulations, and only some of the major points most relevant in the present context will be mentioned. First, under the assumption that the various subpopulations are drawn from a common parent population, Mulaik demonstrates that the factor pattern matrix (which contains the factor loadings) will remain invariant across subpopulations. However this situation will hold only if the same metric is used in the parent and subpopulations, i.e., variance-covariance matrices are mathematically operated upon rather than correlation matrices. If one were to use correlational matrices rather than variance-covariance matrices, it would be necessary to transform the resulting factor pattern matrix into the original common scales by multiplying the factor pattern coefficients by the appropriate original variable standard deviations. In passing it can be noted that such factor invariance depends upon assumptions that may not always be met (see Mulaik, 1972, p. 359).

Most important in the present context is that if subpopulation at occasion $\underline{o}_2$ is from the same parent population as the subpopulation at occasion $\underline{o}_1$, there will be structural invariance, and the only type of change possible is quantitative change (Equation [3]). If, however, the two subpopulations are not from the same parent population (i.e., those same individuals from occasion $\underline{o}_1$ to occasion $\underline{o}_2$ are not really the "same" owing to what may have transpired in the interoccasion interlude), then either structural (Equation [4]) or quantistructural (Equation [5]) change may ensue.

Assuming structural invariance across subpopulations and the use of a common metric (i.e. variance-covariance matrices rather than correlational matrices), as Mulaik notes, it is quite possible to have different interfactor covariances (and correlations) across subpopulations. In fact, Mulaik (1972, p. 358) discusses procedures of comparing different subpopulations in terms of interfactor covariances assuming invariant factor pattern matrices. If one were to define the different subpopulations in terms of the same individuals at various occasions (i.e., longitudinal measures), we now have a means of determining the changes in the interfactor relationships that are discussed in Chapters 5 and 6.

It can be expected that the recent developments discussed above for getting at "real" changes in factor scores, factor loadings, and interfactor relationships, will eventually permit one to operationalize the various change models considered in the present text.

APPENDIX B

The second implicit assumption that is made in the present work is that the factor analytic model is essentially a linear model. Although those adopting multivariate approaches to the structuring of psychological data are generally aware that their techniques are limited to detecting additive and noninteractive relationships, it is true, as Wilson (1973) has pointed out, that this assumption may not be tenable in certain domains, and violations of the linearity assumption are often not fully realized. It is difficult to make a convincing argument for precluding the detection of complex relationships in favor of the mathematically pleasing linear model when in fact it is suspected that the latter will greatly distort the inherent structure in the data. However, as Cattell (1966a), Digman (1966), and Rozeboom (1973) have all pointed out, the linear model will often provide a useful first approximation, or at least is a worthy first candidate, for getting at the latent structure of a data array. Digman (1966) goes somewhat further in his consideration of interaction and nonlinearity in multivariate work, noting several instances where the linear model can readily accommodate to nonlinearity and interaction, and therefore predicting that such a model will continue to play a dominant role in psychological research.

Although the multivariate linear model is attractive, both because of its capability to handle a relatively large number of variables and its mathematical simplicity, this will not justify its blind adoption. When one suspects important interactive relationships which are not amenable to linear analysis in the manner described by Digman (1966), other approaches should be sought, such as those suggested by Wilson (1973). It should be noted, however, that important steps have been taken by proponents of factor analysis to incorporate nonlinear considerations into a basically linear model. Thus Cattell (1966a) has talked about permissive factors, where a certain level on a given factor is required before a second factor comes into play. Such post factor analytic treatment of nonlinear considerations is much in keeping with the view that, although linearity is assumed in identifying factors, this in no way precludes the later mapping out of nonlinear factor interrelationships (Cattell, 1966a; Royce, 1973b). In this way it is possible to, at least post factor analysis, consider conjuntive (both A and B) and disjunctive (either A or B) models, besides the more basic compensatory (trading off relationship) model. In this way, it is possible to partially meet the criticism (Pawlik, 1973; Wilson, 1973) that the factor analytic model is compensatory in nature and unable to deal with conjunctive and disjunctive aspects.

An additional important development to incorporating nonlinear aspects into the basic linear factor analytic model has been made by Cattell (1963). Cattell has elaborated a somewhat analytic modulation theory, where roles and states may modulate, or modify, a whole pattern of personality traits, as determined by both the global or background stimulus conditions, and the more specific focal stimulus situation. In this way, basically linear relationships may become modified in considering a particular total situation.

It will be up to future research to decide the limitations of the present theoretical models in terms of violating the linearity assumption.